Exotic compact objects and how to detect them?

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Outline

- Exotic compact objects as alternative to black holes.
- How to distinguish compact objects and black holes?
- Various observables and implications of them.
- Looking forward.

Reference

- **SC**, Maggio, Mazumdar and Pani, PRD 106, 024041 (2022).
- Nair, **SC** and Sarkar, PRD 107, 124041 (2023); PRD 109, 064025 (2024).
- **SC**, Maggio, Pani and Silvestrini, arXiv: 2310.06023 + in progress.
- Krishnendu, Ghosh, Datta, **SC** and Ajith, Work in progress.

Why Black Holes?

- Black holes can be constructed from normal matter, using simple collapse scenarios.
- Black holes are unique and have universal properties.

[Heusler, Black Hole Uniqueness Theorem (Cambridge University Press)]

- Black holes behave as thermodynamic objects with temperature and entropy — Hint towards quantum gravity. **[Bekenstein, Phys. Rev. D 7, 2333 (1973)]**
- Black holes are stable under all possible perturbations.
- Observation of shadows from Event Horizon Telescope and the ringdown signals from LIGO and VIRGO are definitely consistent with the existence of Black Holes.
- Consistency with general relativity is another story.

But …

- Despite being the simplest objects, there are issues.
- **Singularity**: All black hole spacetimes have a singular region/point ⟶ breakdown of the theory.
- **Loss of Predictability**: Most of the black holes inherit Cauchy horizon ⟶ future cannot be determined.

[Cardoso +, Phys. Rev. Lett. 120, 031103 (2018)]

[Rahman +, JHEP 03, 178 (2019)]

- **Information Loss Paradox**: The existence of thermal radiation results into loss of information.

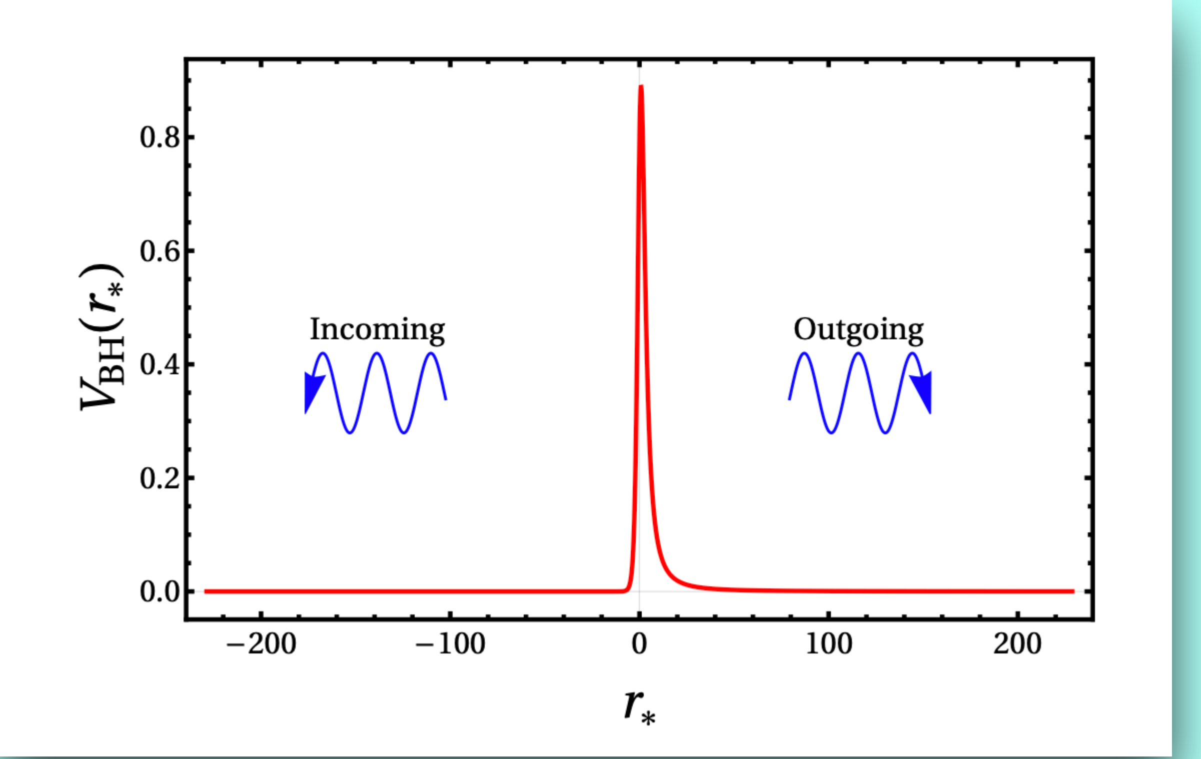
[Hawking, Commun. Math. Phys. 43, 199 (1975)]

[SC and Lochan, Universe 3, 55 (2017)]

- All of these suggest that we may need to look for alternatives — curing these problems and yet remaining consistent with experiments.

Black Hole Hypothesis

- Does the existence of a photon sphere implies the existence of a black hole? **[Cardoso +, Phys. Rev. Lett. 116, 171101 (2016)]**
- The ringdown is governed by the photon sphere alone. **[Cardoso +, Phys. Rev. D 79, 064016 (2009)]**
- Structure beneath the photon sphere can not be probed directly.
- Can such objects exist? What will be their observational properties?



[Figure Courtesy: Biswas +, Phys. Rev. D 106, 124003 (2022)]

Exotic Matter

- Raychaudhuri equation guarantees that normal matter cannot cure singularities $\longrightarrow$ require exotic matter or, quantum effects.

[SC +, Phys. Lett. B 797, 134877 (2019)]

- The consistency with observations, require any alternatives to have

$$\boxed{2M < R < 3M}$$

- The limiting stellar configuration, with normal matter (Buchdahl limit):

$$\boxed{\left(\frac{M}{R}\right) \leq \frac{4}{9}}$$

- Recent shadow measurement argues that Buchdahl limit must be violated $\longrightarrow$ exotic matter is necessary.

[Akiyama et. al., ApJ. Lett. 17, 930 (2022)]

Only Exotic Matter?

- Are these exotic matters stable → ergo-region instability, enhanced superradiant instability, for rotating objects.

[Cardoso +, Phys. Rev. D 77, 124044 (2008)]

- Can quantum effects play any role?

[Abedi +, Phys. Rev. D 96, 082004 (2017)]

- Area quantised black holes are generic predictions of theories of quantum gravity and these have non-trivial physics at horizons.

[Agullo +, Phys. Rev. Lett. 126, 041302 (2021)]

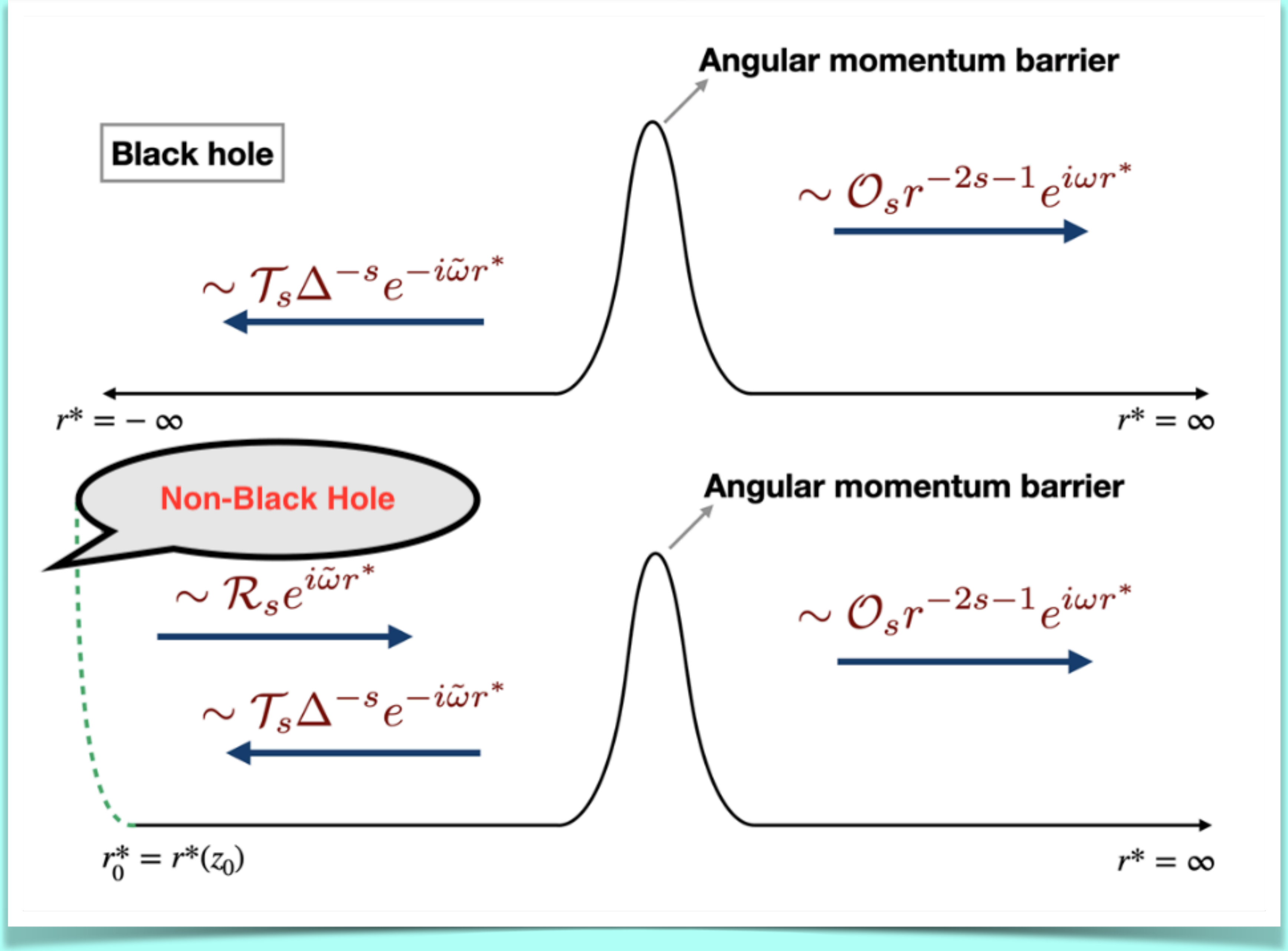
- The basic point is to modify the horizon itself by a reflective membrane as quantum effects are taken into account.

[Cardoso +, JCAP 08, 006 (2019)]

[Maggio +, Phys. Rev. D 102, 064053 (2020)]

[Dey +, Phys. Rev. D 101, 104014 (2020)]

Reflective Horizon — Basics



[Figure Courtesy: Dey +, Phys. Rev. D 103, 084019 (2021)]

Restoring Units: GWs as Magnifying Glass

- Let $\Delta A = \ell_{\rm p}^2$. For non-rotating black holes, $A = 16\pi(GM/c^2)^2$.
- Therefore, the frequency associated with a GW making the above change of area becomes,

$$\nu(\text{in Hz}) = \frac{\ell_p^2}{32\pi(G^2/c^6)Mh} = \frac{c^3}{64\pi^2 GM}$$

- For $M = 10M_{\odot}$, the corresponding frequency becomes $32\ \text{Hz}$!
- Precisely in the LIGO frequency band ! [Agullo +, Phys. Rev. Lett. 126, 041302 (2021)]
- Thus GWs act as a magnifying glass for quantum effects near the horizon. Information about Hawking Radiation is also encoded!

Five Avenues to Probe ECOs

- There are five main avenues to probe quantum effects at horizon aka ECOs.
- These are — (a) QNMs in the ringdown spectrum, (b) Tidal Love Numbers and (c) Tidal Heating associated with in-spiral regime, and environmental effects, due to (d) dark matter and (e) accretion.
- For the QNMs there will be significant deviations at late times.
- The TLNs are expected to have a $(\ln \epsilon)$ behaviour, but appears at 5 pN.

[Cardoso +, Phys. Rev. D 95 084014 (2017)]

- The Tidal Heating starts contributing at 2.5 pN.

[Datta +, PRD 101, 044004 (2020)]

- Environmental effects are omnipresent and can distinguish ECOs from BHs, either through dark matter or through accretion.

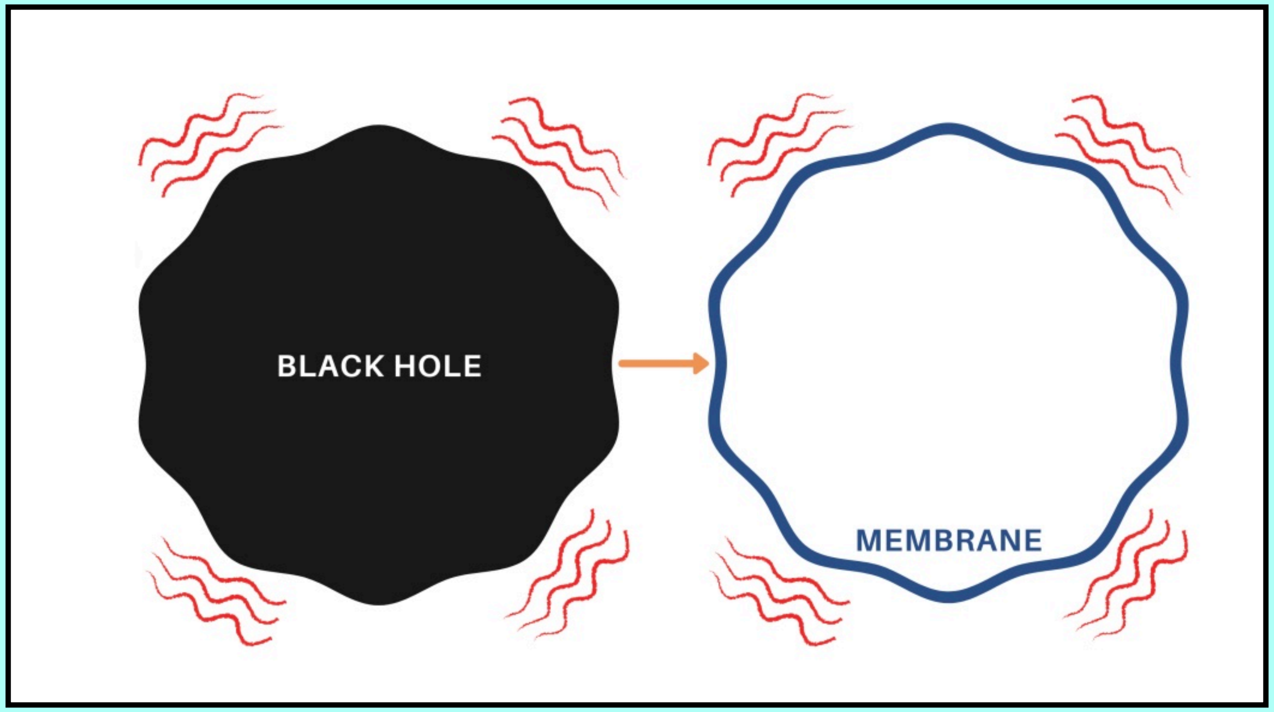
[Mitra +, arXiv: 2312.06783]

[Chowdhury +, arXiv: 2405.04006]

QNMs

Membrane Paradigm and GW

- Replacing the black hole horizon by a membrane is a natural assumption, giving rise to the membrane paradigm.
- Any reflective boundary, close to the horizon, arising due to some exotic compact object, can also be described by a similar membrane fluid.



$$T_{ab} = \rho u_a u_b + (p - \zeta\Theta)\,\gamma_{ab} - 2\eta\sigma_{ab}$$

$$[[K_{ab} - h_{ab}K]] = -8\pi T_{ab}$$

[Price and Thorne, Phys. Rev. D 33, 915 (1986)]

[Figure Courtesy: Maggio +, Phys. Rev. D 102, 064053 (2020)]

[SC +, Work in Progress]

“Quantum” Matter = Geometry

- The properties of matter get related to the geometry by the junction conditions: $\boxed{[[K_{ab} - K h_{ab}]] = -8\pi \langle \hat{T}_{ab} \rangle}$ and $\boxed{[[h_{ab}]] = 0}$ on $\boxed{R = r_+ + \langle \hat{\epsilon} \rangle}$.
- These must be perturbed due to perturbation of the metric and the governing equation will become [SC +, Phys. Rev. D 106, 024041 (2022)]

$$\boxed{\delta K_{ab} - K \delta h_{ab} = -8\pi \langle \delta \hat{T}_{ab} \rangle}$$

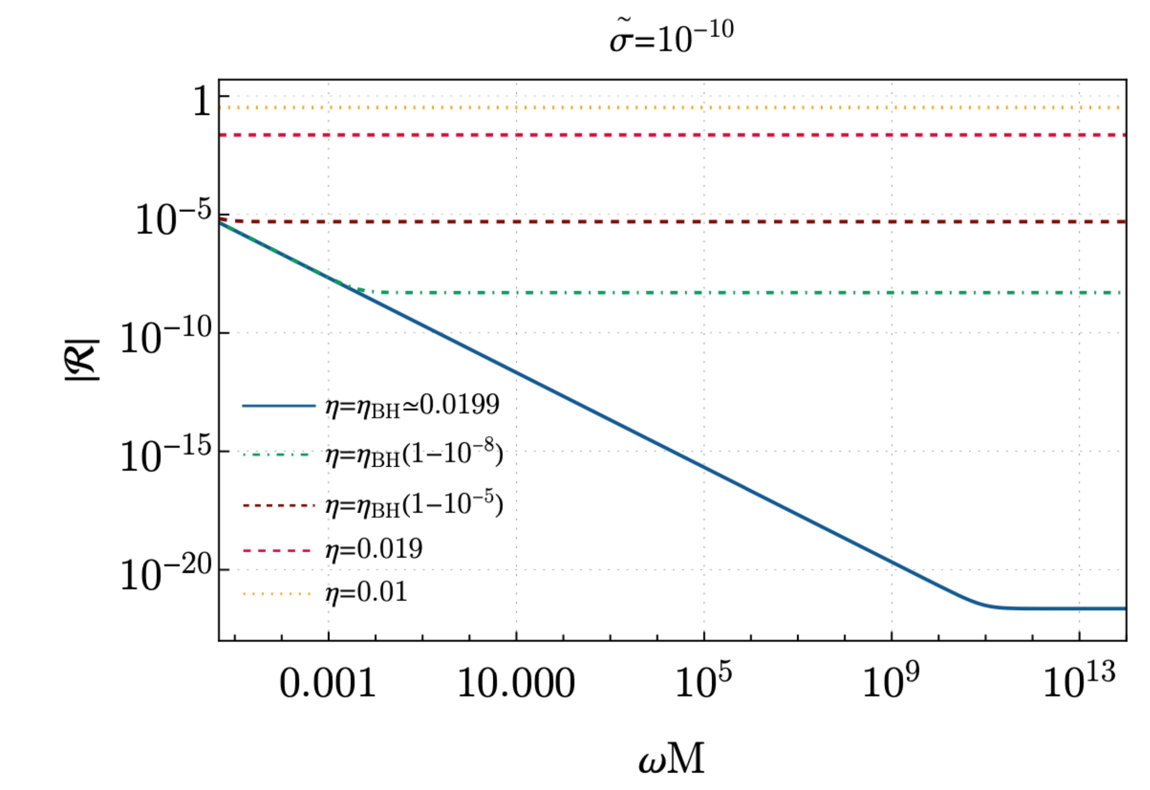
- For simplicity we will consider axial gravitational perturbation.

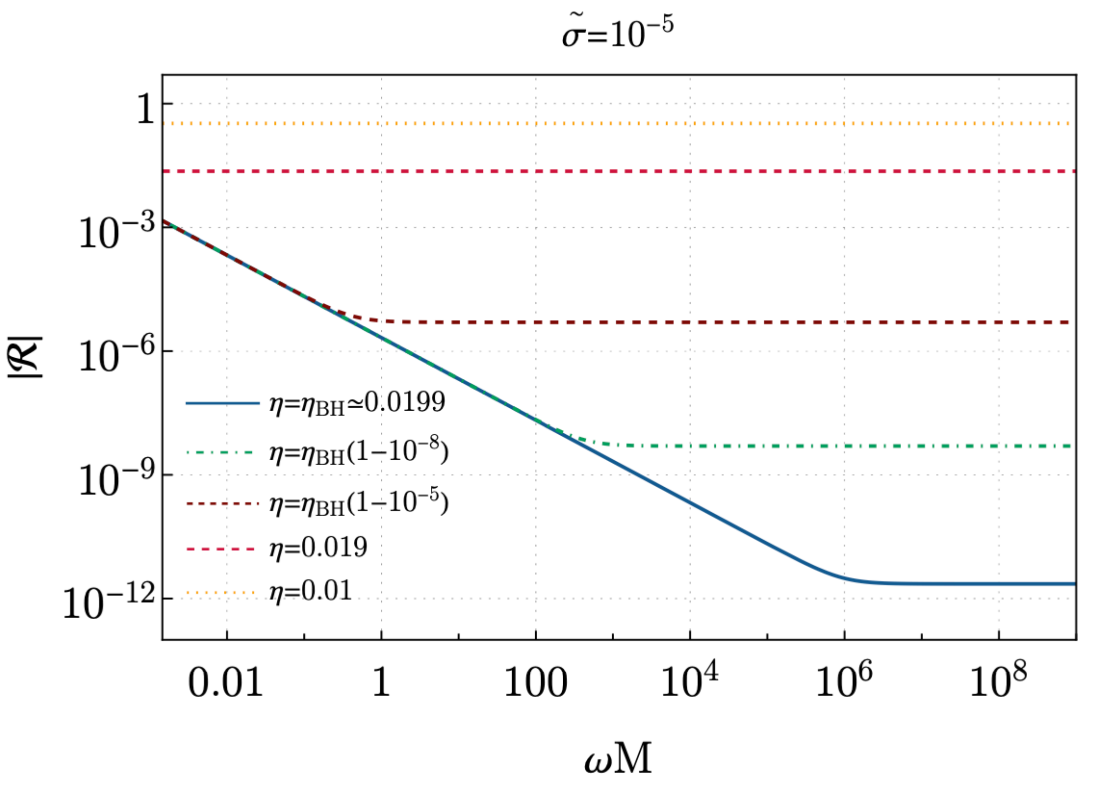
$$\boxed{\begin{aligned} i\omega\psi(R) = \frac{\eta}{(\rho_0 + p_0)\sqrt{f(R)}} \Bigg[& V_{\rm axial}(R)\psi(R) - \frac{1}{R}\frac{d\psi(R)}{dx}\left[Rf'(R) - 2f(R)\right] \\ & - \frac{4f(R)}{R}\left(\frac{d\psi(R)}{dx} + \frac{f(R)}{R}\psi(R)\right)\left(1 + \frac{4\pi\rho_0 R}{\sqrt{f(R)}}\right)\Bigg] . \end{aligned}}$$

Reflecting "quantum" membrane

[SC +, Phys. Rev. D 106, 024041 (2022)]

- The boundary condition allows existence of ingoing as well as outgoing waves near the membrane, such that, $$\psi_{\mathrm{M}} = e^{-i\omega x} + \mathcal{R} e^{i\omega x}$$
- In appropriate limits, $$|\mathcal{R}|^2 \sim \left(\frac{1-\eta/\eta_{\mathrm{BH}}}{1+\eta/\eta_{\mathrm{BH}}}\right)^2 + \frac{16384\,[\ell(\ell+1)-3]^2\,\pi^3\eta^4\tilde{\sigma}^2}{(1+\eta/\eta_{\mathrm{BH}})^4\,\omega^2 M^2}$$

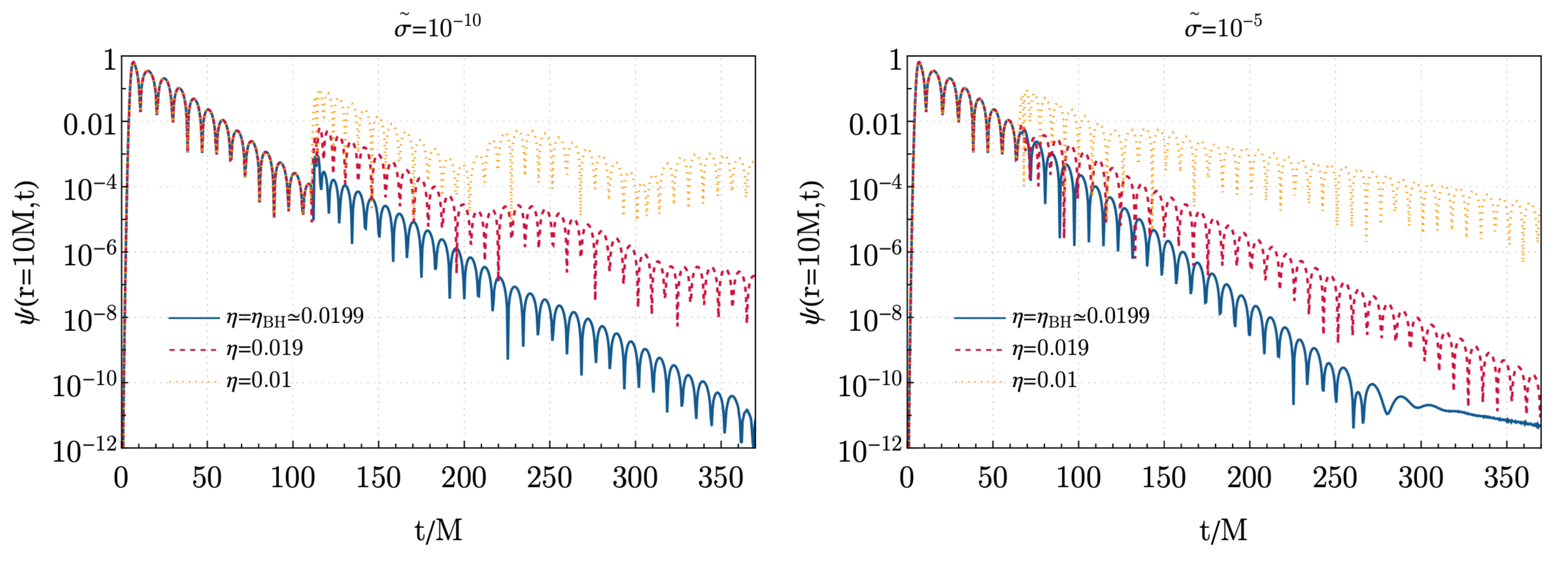




Ringdown Waveform

- As the effective classical membrane nears the horizon, there are pronounced echoes.
  [SC +, Phys. Rev. D 106, 024041 (2022)]

- The time delay is consistent with size of the membrane.



Tidal Love Numbers

Tides are Everywhere

- Tides are ubiquitous and captures the true nature of gravity, as it depends on the Riemann.
- Deformation produced by the tides depends on the detail of the constituent of the object being deformed.
- What about black holes? Can they be deformed?
- In the context of GW, the effects due to tidal deformation appears at 5 pN, thus unless the deformation is large, detection can be challenging.
- What happens if we consider objects as compact as BHs, but with reflective surface? Can they be deformed?
- Does deformation depends on asymptotic properties of spacetime? Can a Schwarzschild-de Sitter BH be deformed?

Newtonian Deformation

- Newtonian gravity is solely governed by the potential.
- The tidal field arises from double derivative of the potential.

$$\boxed{\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}.}$$

- The total potential will consist of potential due to the external tidal field and the potential of the deformed object. **[Thorne, Phys. Rev. D 58, 124031 (1998)]**

$$\begin{aligned} \frac{(1-g_{tt})}{2} &= -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) \\ &\quad + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O\left(r^3\right), \end{aligned}$$

- Quadrupole moment of the deformed object arises due to tidal field.

$$\boxed{Q_{ij} = -\lambda \mathcal{E}_{ij}.}$$

$$\boxed{k_2 = \frac{3}{2} G \lambda R^{-5}.}$$

Gauge Invariant Tidal Love Number

- For the Love number to be gauge invariant, one solves the Teukolsky equation for Weyl scalars and impose boundary conditions near the horizon.

[Le Tiec +, Phys. Rev. D 103, 084021 (2021)]

$$\lim_{c\to\infty} c^2\psi_0 = \sum_{\ell m} \alpha_{\ell m}(t)\, r^{\ell-2}\left[1 + 2k_{\ell m}\left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}(\theta,\varphi)\,,$$

$$F_{\ell m}(\omega) = 2\,k_{\ell m} + i\omega\tau_0\,\nu_{\ell m} + \mathcal{O}(\omega^2)\,.$$

[Chia, Phys. Rev. D 104, 024013 (2021)]

- For Kerr black hole, the Love number becomes

$$k_{lm} = (am)^2\,k^{(0)}_{lm} + am\omega\,k^{(1)}_{lm} + \mathcal{O}(M^2\omega^2)$$

[Bhatt +, Phys. Rev. D 108, 084013 (2023)]

[Bhatt +, arXiv: 2406.09543]

Compact Objects can be deformed

- Compact Objects having non-zero reflectivity, in general, have non-zero Love number. [SC, Maggio, Silvestrini and Pani, arXiv: 2310.06023]

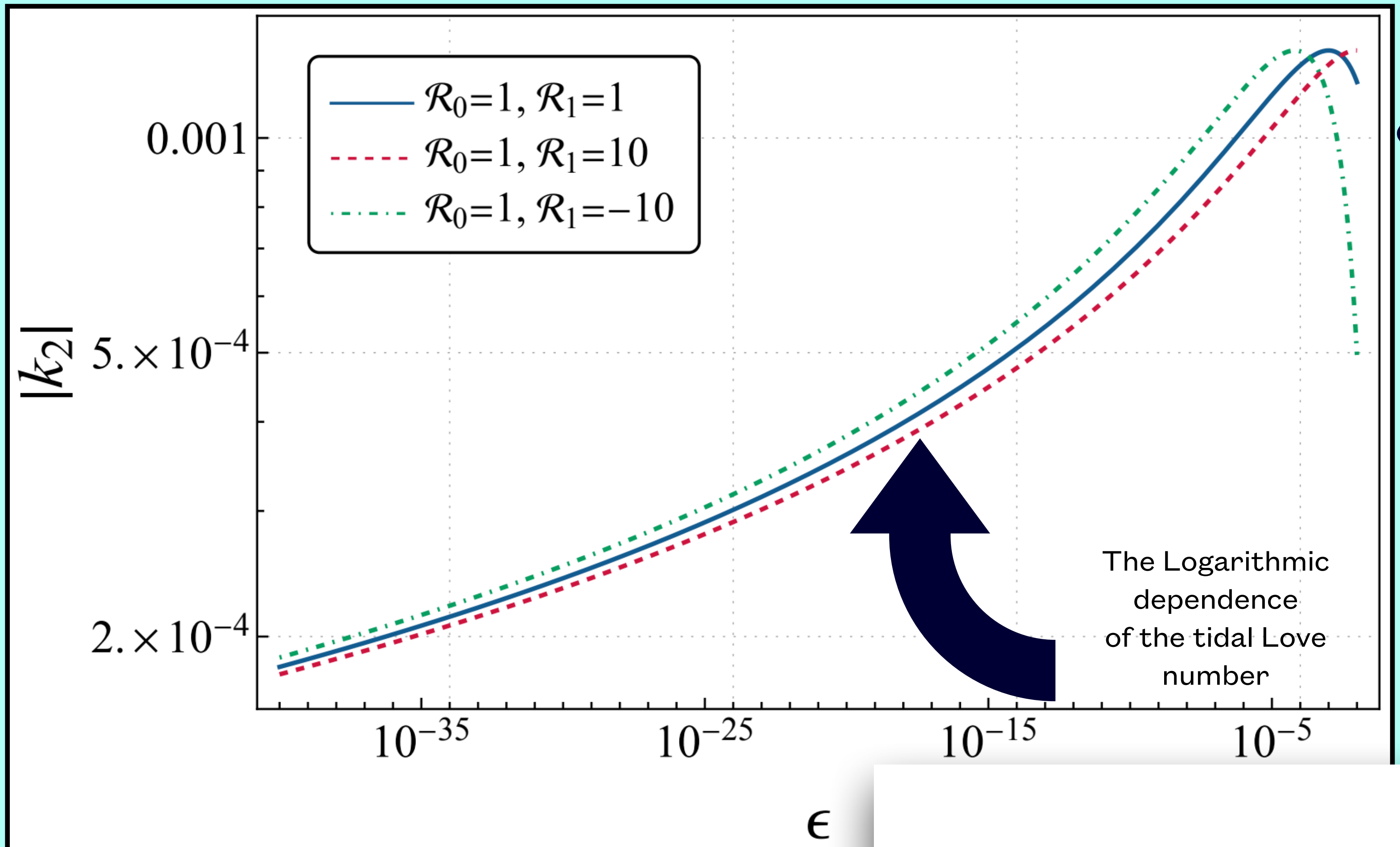
$$k_2 = \mathrm{Re}\left[\left(\frac{iM\omega}{30}\right)\left(1+16M^2\omega^2\right)\left(1+4M^2\omega^2\right) \times \left\{\frac{1-\frac{\mathcal{B}}{\mathcal{A}}\Gamma_1}{1+\frac{\mathcal{B}}{\mathcal{A}}\Gamma_1}\right\}\right],$$

$$= \frac{2}{15}\mathrm{Re}\left[\frac{1}{-2\mathcal{R}_1 + \{7+16i\pi+8(\epsilon+\ln\epsilon)\}}\right]$$

- Reflectivity is defined in terms of the Detweiler function, which can be related to the Teukolsky function.

- It turns out that the Love number is non-zero, if and only if

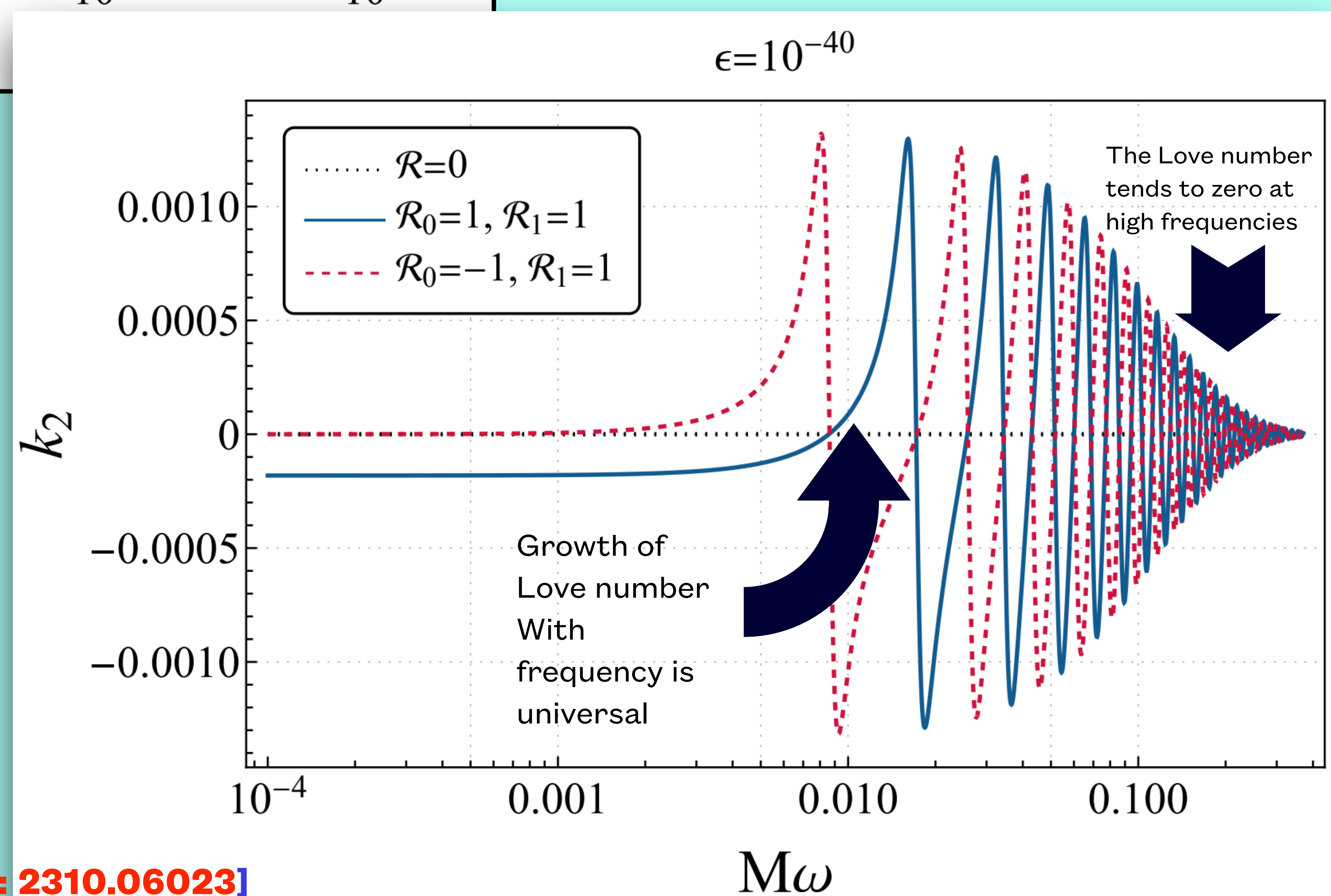
$$\boxed{\mathcal{R}(\omega) = 1 + iM\omega\mathcal{R}_1}$$



The frequency dependent Part of the reflectivity takes over

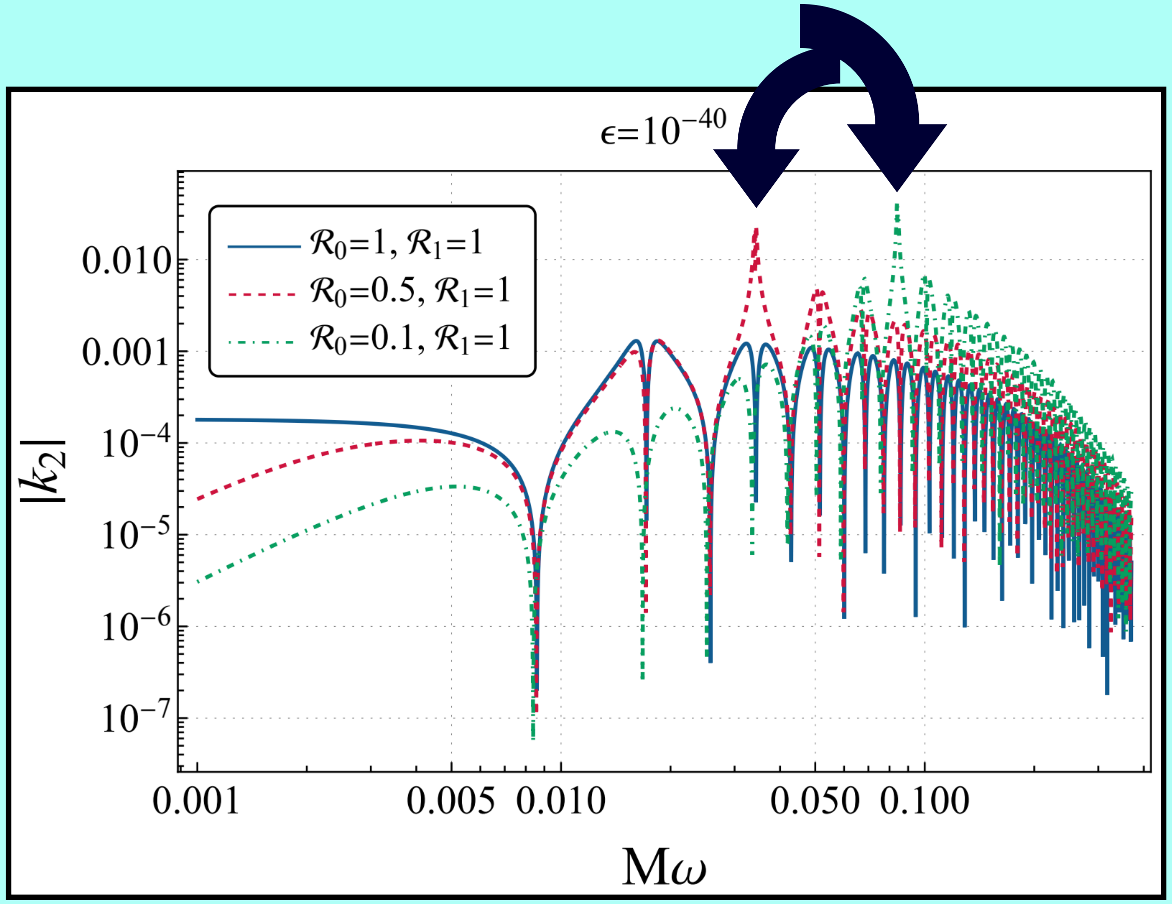
- The Logarithmic behaviour is specific to unit reflectivity.
- Love number heavily depends on the phase of the reflectivity.

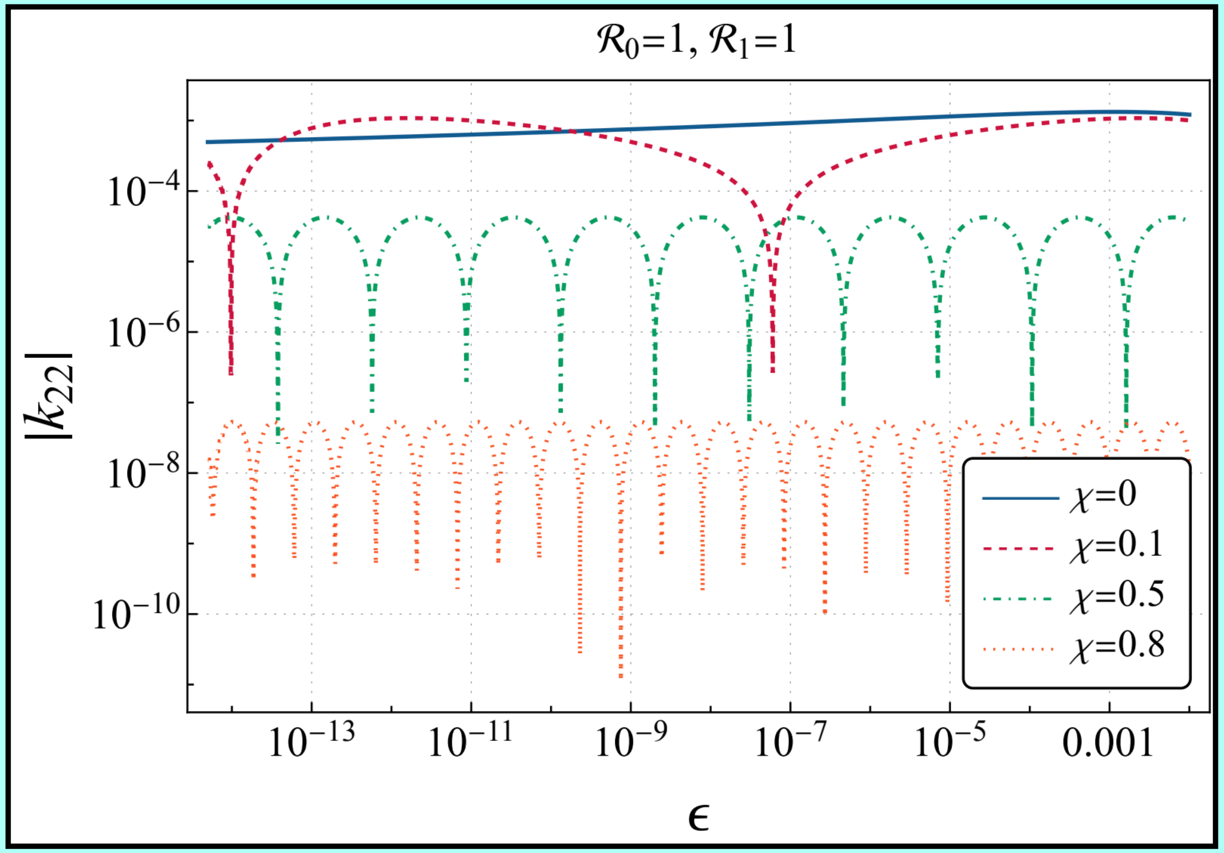
- The tidal Love number grows with frequency.
- At higher frequencies the Love number tends to zero → the compact object effectively behaves as a BH.

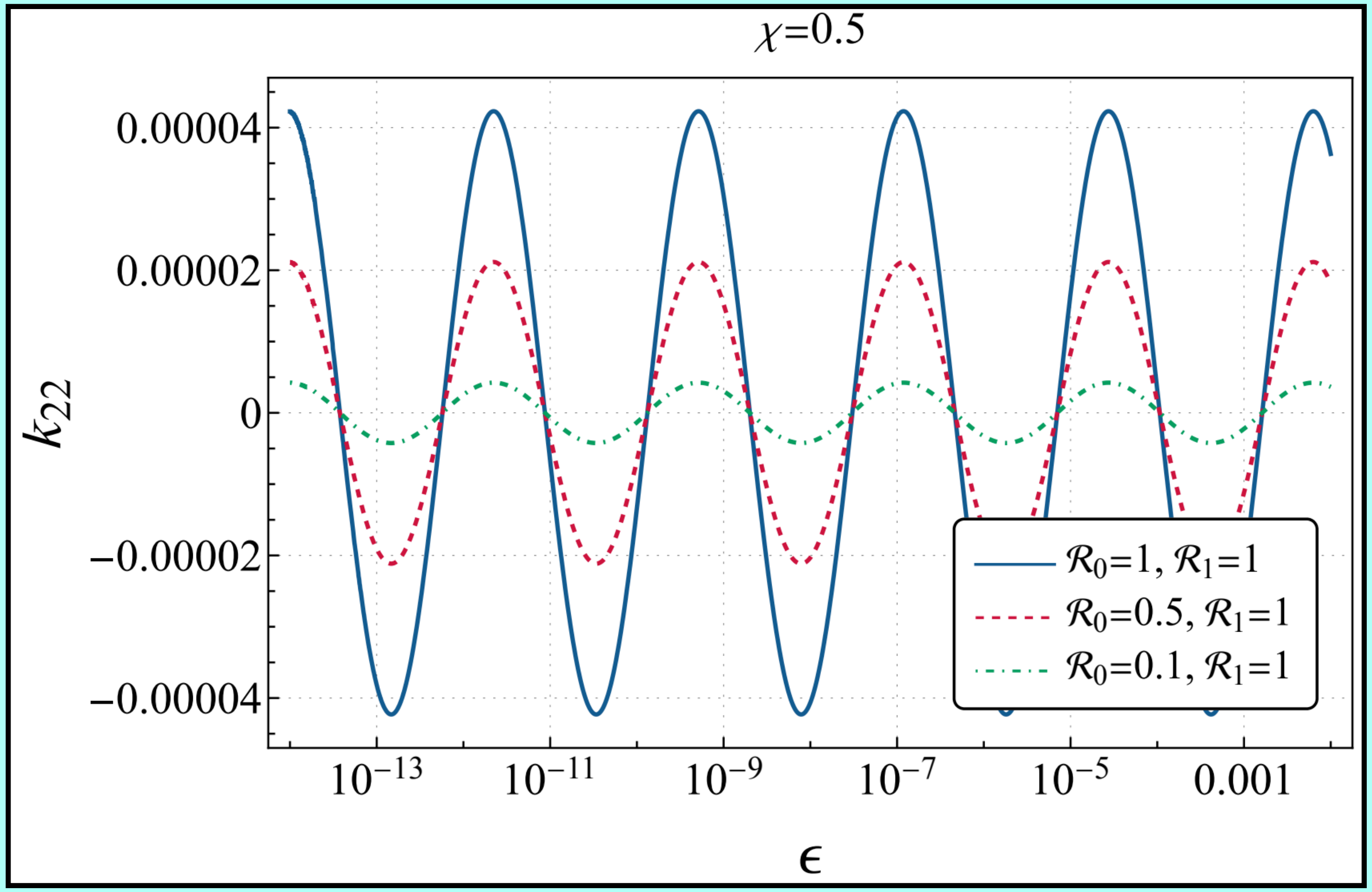


[SC, Maggio, Silvestrini and Pani, arXiv: 2310.06023]

Resonances at QNM frequencies







- There are resonances in the Love number at QNM frequencies.
- Increasing rotation and decreasing reflectivity decreases the Love number drastically.

[SC, Maggio, Silvestrini and Pani, arXiv: 2310.06023]

Consequences

- First of all, the TLNs are non-zero for quantum black holes or, for ECOs, but depends heavily on the model of interest.

- The characteristic frequencies for typical inspirals are well within either LIGO or LISA sensitivity band and hence it is possible to detect them.

- But, the numerical values of the TLNs are very small, $\mathcal{O}(10^{-2})$. Thus detection beyond the noise can be challenging.

- The scaling of the TLNs with $\ln\epsilon$ is universal, but the fact that for BHs also dynamical TLNs can be non-zero makes the distinction between BHs and ECOs challenging.

[SC +, Work in Progress]

Tidal Heating

What is Tidal Heating

- As two compact objects in-spiral around each other, the tidal field of one affects the other, thereby introducing gravitational perturbations.
- A part of these perturbations travel to infinity, while a part travels within the horizon, changing the horizon area, mass and angular momentum.

[Alvi, PRD 64, 104020 (2001)]

[Chatziioannou +, PRD 87, 044022 (2013)]

- These modifications in geometrical properties of compact objects is referred to as tidal heating.
- Since the tidal heating phenomenon involves knowledge about the nature of the horizon, it is going to capture quantum effects/nature of exotic compact objects.

[Datta +, PRD 101, 044004 (2020)]

[Saketh +, PRD 107, 084006 (2023)]

Rate of change of area and mass

- Perturbations modify the null generators and hence the shear. [Hawking and Hartle, Commun. Math. Phys. 27, 283 (1972)]

$$\frac{dA_{\rm BH1}}{dt} = \frac{2}{\kappa_{+1}} \int |\sigma_{\rm HH}|^2 \sqrt{h^{(2)}} d^2x = \frac{2(r_{+1}^2 + a_1^2)}{\kappa_{+1}} \int |\sigma_{\rm HH}|^2 d\Omega \ ,$$

- The shear is related to the gravitational perturbation. Then using first law of black hole mechanics the corresponding mass increase rate can be determined.

$$\frac{dM_1}{dt} = \left(\frac{dE}{dt}\right)_{\rm N} \left(\frac{M_1}{M}\right)^3 \frac{v^5}{4} \left\{-\chi_1 \left(\hat{\mathbf{L}}_{\rm orb} \cdot \hat{\mathbf{J}}_1\right) + 2\frac{v^3}{M}\left[r_{+1} + \frac{Q}{2M_1}\right]\right\} \times \left[1 + 3\chi_1^2 + \frac{Q}{M_1^2}\left(2 + 3\chi_1^2 + \frac{Q}{M_1^2}\right)\right] \ ; \quad \left(\frac{dE}{dt}\right)_{\rm N} \equiv \frac{v^3}{M}\left(\frac{dJ}{dt}\right)_{\rm N}$$

[SC +, PRD 104, 104001 (2021)]

What happens for Area Quantized BHs?

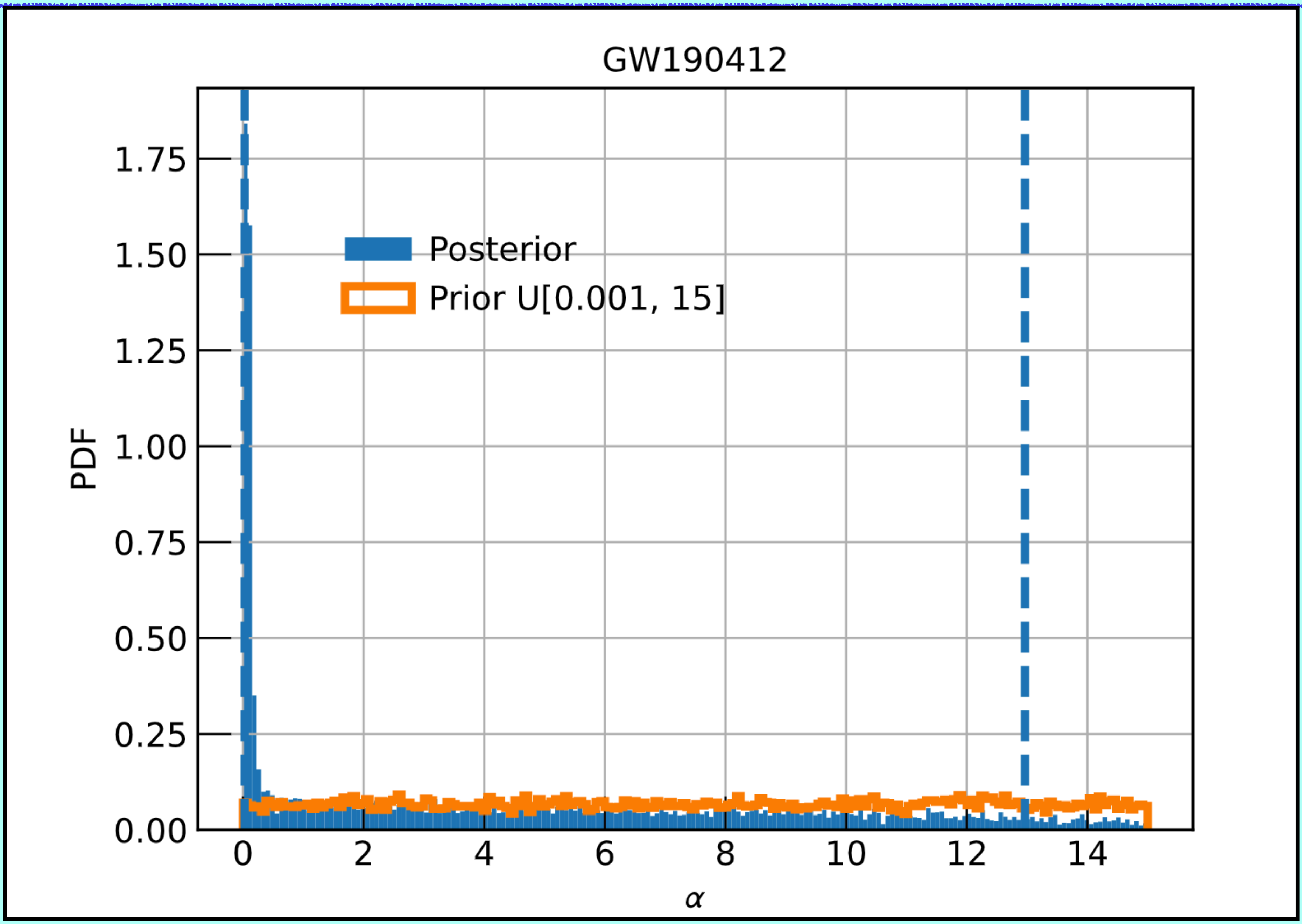
- For area-quantized BHs, the phase in the GW signal gets modified

$$\begin{aligned}\Psi_{\mathrm{THQBH}} = & \frac{3}{128\nu}\left(\frac{1}{v}\right)^{5}\Bigg[-\frac{10}{9}v^{5}\Psi_{5}\left(3\log\left(v\right)+1\right) \\ & -\frac{5}{168}v^{7}\Psi_{5}\left(952\nu+995\right) \\ & +\frac{5}{9}v^{8}\left(3\log\left(v\right)-1\right)\left(-4\Psi_{8}+\Psi_{5}\psi_{\mathrm{SO}}\right)\Bigg]\end{aligned}$$

[Datta and Phukon, Phys. Rev. D 104, 124062 (2021)]

- The Ψ_5 and Ψ_8 terms depend on the reflectivity of the area-quantized BH and hence on the parameter α.
- Also QBH absorbs only if the frequency exceeds a minimum value $2\Omega_{\mathrm{H}}$.
- It is also bounded above by the ISCO frequency.

Constraint on area quantized BH



[Krishnendu + Work in progress]

Consequences

- The tidal heating appears in the phase of the GW waveform at a lower pN order (2.5 pN with rotation and 4 pN without rotation). Thus the effect of reflectivity can be directly observed. What about the 3.5 pN term?

  [Chatziioannou +, Phys. Rev. D 94, 084043 (2016)]

- For area quantised black holes, better results are expected if one can probe beyond the ISCO frequency. Thus study of the plunge phase will be important.

  [Mukherjee and SC, Class. Quant. Grav. 40, 145013 (2023)]

Environmental Effects

Environment with ultra-light dark matter

- All compact objects live in an environment consisting of dark matter.
- If the dark matter consist of ultra-light scalar particles, then in-spiral of compact objects in such environment can distinguish between quantum and classical BHs. **[Cardoso and Maselli, arXiv: 1909.05870]**

[Vicente and Cardoso, Phys. Rev. D 105, 083008 (2022)]

- From scattering of scalar field from a rotating compact object, it follows that the motion of a compact object in the dark matter environment will involve accretion as well as dynamical friction as two dominating effects.

[Traykova +, Phys. Rev. D 104, 103014 (2021)]

$$\dot{M} \approx \rho A_{\rm h}\left(\frac{e^{-\pi\eta}\pi\eta}{\sinh(\pi\eta)}\right)\mathrm{Re}\left(\frac{1-\mathfrak{R}}{1+\mathfrak{R}}\right) \qquad \eta \equiv -\alpha_{\rm g}(1+v^2)/v \qquad \alpha_{\rm g} \equiv \gamma\mu M$$

$$\boldsymbol{F} = -\frac{4\pi\rho\boldsymbol{v}}{\mu^2 v}\left\{\eta^2\mathrm{Re}\left[\psi(1+l_{\max}+i\eta)-\psi(1+i\eta)\right]+\frac{\omega k_\infty A_+}{4\pi}\left(\frac{\pi\eta e^{-\pi\eta}}{\sinh(\pi\eta)}\right)\mathrm{Re}\left(\frac{1-\mathfrak{R}}{1+\mathfrak{R}}\right)\right\}$$

[Mitra +, arXiv: 2312.06783]

Effect on GW phase

- The accretion rate in the rest frame of dark matter is: $\dot{M}' = \dot{M}/\gamma$
- The dynamical friction in that frame corresponds to: $F'_i = F_i + \dot{M} v^i$
- Thus dynamical friction does **not** depend on the reflectivity of the compact object, but the accretion rate does. This leads to a change in the phase of the GW signal.

[Mitra +, arXiv: 2312.06783]

EMRI

$$\varphi^{\rm vac} \approx \frac{5q}{16v^6}\Delta v \sim 8\times 10^5 \frac{f}{1\,\mathrm{mHz}}\frac{T_{\rm obs}}{4\,\mathrm{yr}},$$
$$\varphi^{\rm df} \sim -3\times 10^4 \frac{\rho_0 M_{\rm T}^2}{10^{-7}}\left(\frac{\mu M_{\rm T}}{0.1}\right)^2\left(\frac{10^6 M_\odot}{M_{\rm T}}\right)^{\frac{13}{3}}\left(\frac{1\,\mathrm{mHz}}{f}\right)^{\frac{10}{3}}\frac{T_{\rm obs}}{4\,\mathrm{yr}},$$
$$\varphi^{\rm acc} \sim -2\times 10^4 \mathcal{T}_2 \frac{\rho_0 M_{\rm T}^2}{10^{-7}}\left(\frac{10^6 M_\odot}{M_{\rm T}}\right)^{\frac{8}{3}}\left(\frac{1\,\mathrm{mHz}}{f}\right)^{\frac{5}{3}}\frac{T_{\rm obs}}{4\,\mathrm{yr}}.$$

Supermassive BH Binary

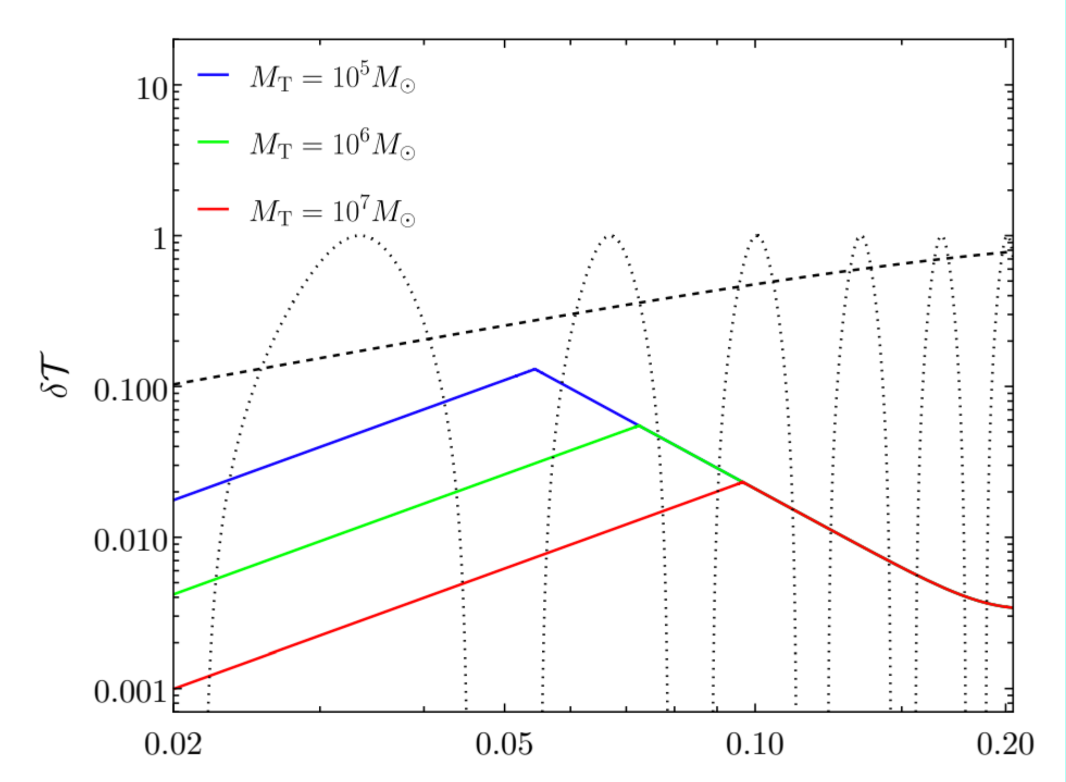
$$\varphi^{\rm vac} \approx \frac{v_{\rm f}^5 - v_{\rm i}^5}{4(v_{\rm i}v_{\rm f})^5},$$
$$\varphi^{\rm df} \approx -\frac{25\pi(v_{\rm f}^{18} - v_{\rm i}^{18})}{1728(v_{\rm i}v_{\rm f})^{18}}\rho_0 M_{\rm T}^2(\mu M_{\rm T})^2,$$
$$\varphi^{\rm acc} \approx -\frac{25\pi\mathcal{T}(v_{\rm f}^{13} - v_{\rm i}^{13})}{52(v_{\rm i}v_{\rm f})^{13}}\rho_0 M_{\rm T}^2(1+\sqrt{1-\chi^2}),$$

Constraints from LISA

- Using the sensitivity of LISA one can find out the minimum dark matter density for environmental effects to be observable.
- Further, from supermassive BH binaries one can constrain the rate of accretion, and hence the reflectivity, such that one can distinguish not only classical and quantum BHs, but also models of quantum BHs.

$$\boxed{\frac{\delta\varphi}{\varphi^{\mathrm{vac}}} \sim 5\times 10^{-3}\left(\frac{10^2}{\mathrm{SNR}}\right)\left(\frac{4\,\mathrm{yr}}{T_{\mathrm{obs}}}\right)}$$

$$\boxed{\rho_0 \overset{(q\gg 1)}{\gtrsim} 10^{-3}\,\mathrm{g/cm^3}\left(\frac{0.1}{\mu M_{\mathrm{T}}}\right)^2\left(\frac{M_{\mathrm{T}}}{10^6 M_\odot}\right)^{\frac{7}{3}}\left(\frac{f}{1\,\mathrm{mHz}}\right)^{\frac{13}{3}}\frac{10^3}{\mathrm{SNR}}}$$



[Mitra +, arXiv: 2312.06783]

Consequences

- The above analysis assumes homogeneous distribution of dark matter. However, that is not the case in general. There can be dark matter spikes, dark matter distributed with specific radial profiles, etc. The study has to include such more realistic situations.

  [Ferreira, arXiv: 2005.03254]

- Environmental effects can also make the distinction between BHs and ECOs more difficult. For example, Schwarzschild BH surrounded by DM has non-zero tidal Love number, alike ECOs.

  [Cardoso +, arXiv: 2109.00005]

  [SC +, Work in Progress]

- Accretion of plasma also has the potential to distinguish BHs from ECOs.

  [Chowdhury +, arXiv: 2405.04006]

Conclusion

- ECOs generically impose reflective nature on the black hole horizon.
- There are echoes in the ringdown signal, which depends on the reflectivity.
- The in-spiral part of the GW waveform also gets affected — tidal Love number of ECOs are non-zero, and scales logarithmically. Tidal heating seems more promising in constraining ECOs.
- Environments involving ultra-light dark matter is also an excellent probe to find out ECOs through LISA.

Thank You