Fate of the Fixed Points of the Quartic Couplings in the Inert Models

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From Big Bang to Now : A Theory-Experiment Dialogue

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*Fate of the **Fixed Points** of the **Quartic Couplings** in the **Inert Models***

Outline of the Talk

- Fixed Points and Landau Poles
- (Not) In Standard Model
- In Inert Extensions (IS, ITM, IDM)

Based on ONGOING work with
Dr. Priyotosh Bandyopadhyay

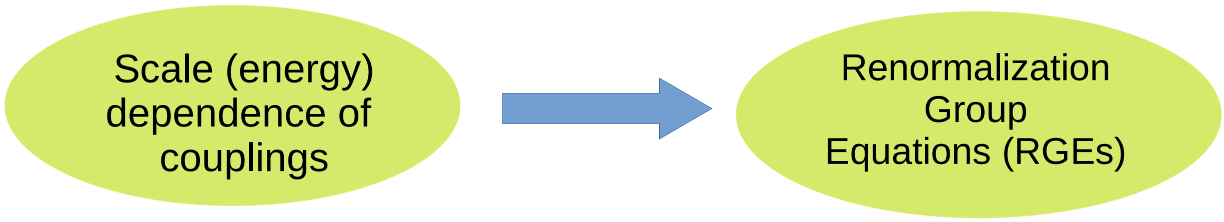
Fixed Points and Landau Poles

- *Introduction: RGEs*
- *Example: ϕ^4- theory*
- *Patterns ?*

Renormalization Group Equations

Renormalization Group approach is a powerful method to study how physical systems behave under different scales.

In QFT, the scale means energy scale



Renormalization Group Equations

For the classical Lagrangian, $\mathcal{L} = \mathcal{L}(\phi_i, \psi_j, A_k)$

there could be a number of couplings given by,

$$g_1, g_2, \ldots, \ \lambda_1, \lambda_2, \ldots, \ y_1, y_2, \ldots$$

quantum corrections

Their scale dependence is given by the set of coupled differential equations,

$$\mu \frac{d}{d\mu} g_i(\mu) = \beta_{g_i}(g_1, g_2, \ldots, \lambda_1, \lambda_2, \ldots, y_1, y_2, \ldots)$$

$$\mu \frac{d}{d\mu} \lambda_j(\mu) = \beta_{\lambda_j}(g_1, g_2, \ldots, \lambda_1, \lambda_2, \ldots, y_1, y_2, \ldots)$$

$$\mu \frac{d}{d\mu} y_k(\mu) = \beta_{y_k}(g_1, g_2, \ldots, \lambda_1, \lambda_2, \ldots, y_1, y_2, \ldots)$$

$\beta_{g_i}, \beta_{\lambda_j}, \beta_{y_k}$

are called the β-functions

Here μ is the energy scale

Renormalization Group Equations

Fixed Points

The *β*-functions vanish → The theory become ***scale-invariant***

$$\beta_{g_i} = 0, \quad \beta_{\lambda_j} = 0, \quad \beta_{y_k} = 0.$$

Different Types of fixed points:

Infra-red fixed point

Ultra-violet fixed point

Gaussian fixed point

Imortant in understanding
Conformal symmetries

If theory is scale-invariant, particle interaction strength become independent of their energy

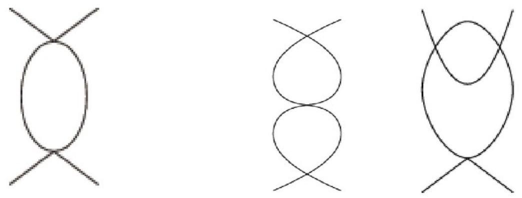
Fixed points in ϕ^4- theory

The Lagrangian is given by,

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

The quartic coupling λ evolves according to the equation

$$\mu \frac{d\lambda(\mu)}{d\mu} = \beta_\lambda(\lambda)$$



The β-function is calculated as a perturbative series,

$$\beta_\lambda(\lambda) = \mu \frac{d\lambda}{d\mu} = \sum_{k=2}^{\infty} c_k \lambda^k, \qquad \text{or,} \qquad \beta_a(a) = a \sum_{\ell=1}^{\infty} b_\ell a^\ell .$$

where, $a \equiv \dfrac{\lambda}{(4\pi)^2}$

The coefficients c_k and b_ℓ are determined via **loop calculations.**

Fixed points in ϕ^4- theory

The n-loop β-function is given by,

$$\beta_a^{(n)}(a) = a\sum_{\ell=1}^{n} b_\ell a^\ell.$$

polynomial
of degree
n + 1

real roots
⬇
fixed points

We can factor out a^2

$$\beta_a^{(n)}(a) = a^2\left(\sum_{\ell=1}^{n} b_\ell a^{\ell-1}\right) \equiv a^2\tilde{\beta}^{(n)}(a).$$

a = 0   (λ = 0)
is a root.
Gaussian fixed
point

polynomial
of degree
n - 1

Fixed points in ϕ^4- theory

The coefficients are given by,

$$b_1 = 3, \quad b_2 = -\frac{17}{3}, \quad b_3 = \frac{145}{8} + 12\zeta_3,$$

$$b_4 = -\frac{3499}{48} - 78\zeta_3 + 18\zeta_4 - 120\zeta_5,$$

Robert Shrock
PRD 107, 056018 (2023)

$$b_5 = \frac{764621}{2304} + \frac{7965}{16}\zeta_3 - \frac{1189}{8}\zeta_4 + 987\zeta_5 + 45\zeta_3^2 - \frac{675}{2}\zeta_6 + 1323\zeta_7,$$

ζ_s is the Riemann zeta function

$$\zeta_s = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

n	1	2	3	4	5	6	7
b_n	3	-5.6666	32.5497	-271.606	2848.57	-34776.13	474651.0
a_{FP}	Nil	0.5294	Nil	0.2333	Nil	0.1604	Nil

Landau poles in ϕ^4- theory

At a Landau pole the *perturbative approach becomes completely unreliable*.

The Landau pole in the one loop can be obtained by solving the one loop RG equation.

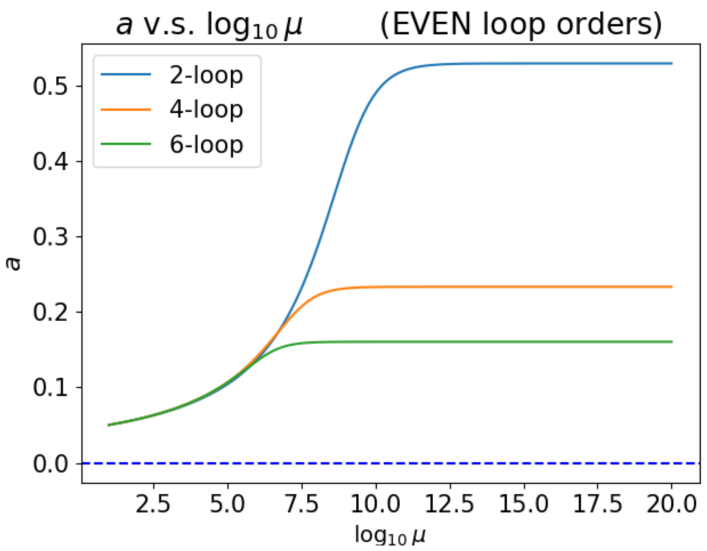
$$\mu \frac{da}{d\mu} = b_1 a^2, \quad \implies \quad \int_{a_0}^{a} \frac{da'}{(a')^2} = b_1 \int_{\mu_0}^{\mu} \frac{d(\mu')}{\mu'}, \quad \implies \quad a(\mu) = \frac{a_0}{1 - a_0 b_1 \ln(\frac{\mu}{\mu_0})}.$$

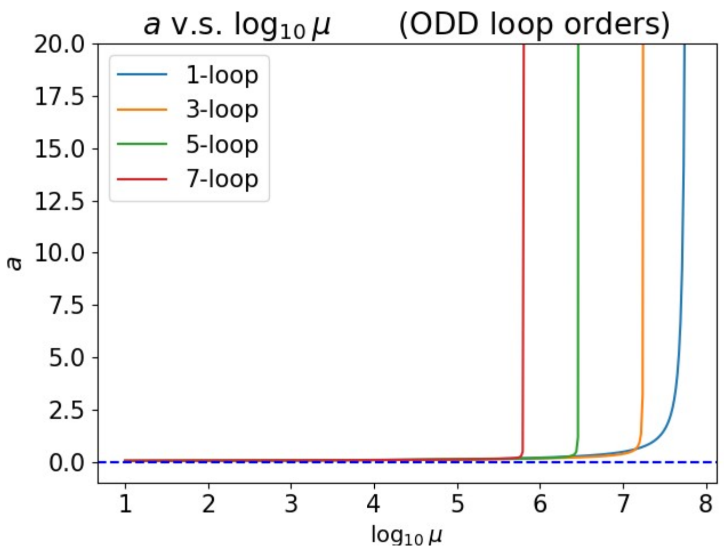
In terms of λ we have,

$$\lambda(\mu) = \frac{\lambda_0}{1 - \frac{3\lambda_0}{16\pi^2} \ln(\frac{\mu}{\mu_0})},$$ where $\lambda_0 = \lambda(\mu_0)$, at some reference scale μ_0.

At the finite energy scale $\mu = \mu_0 \exp(\frac{16\pi^2}{3\lambda_0})$ we have **Landau pole**.

Fixed points and Landau poles in ϕ^4- theory





The $\beta^{(n)}$, for even n, up to $n = 6$, exhibits fixed points.

But the positions of the fixed points are different.

The $\beta^{(n)}$, for odd n, up to $n = 7$, exhibits a Landau pole.

But the positions of the Landau poles are different.

What about Standard Model ?

- *Couplings of SM*
- *RG Evolution*
- *Fixed Points ?*

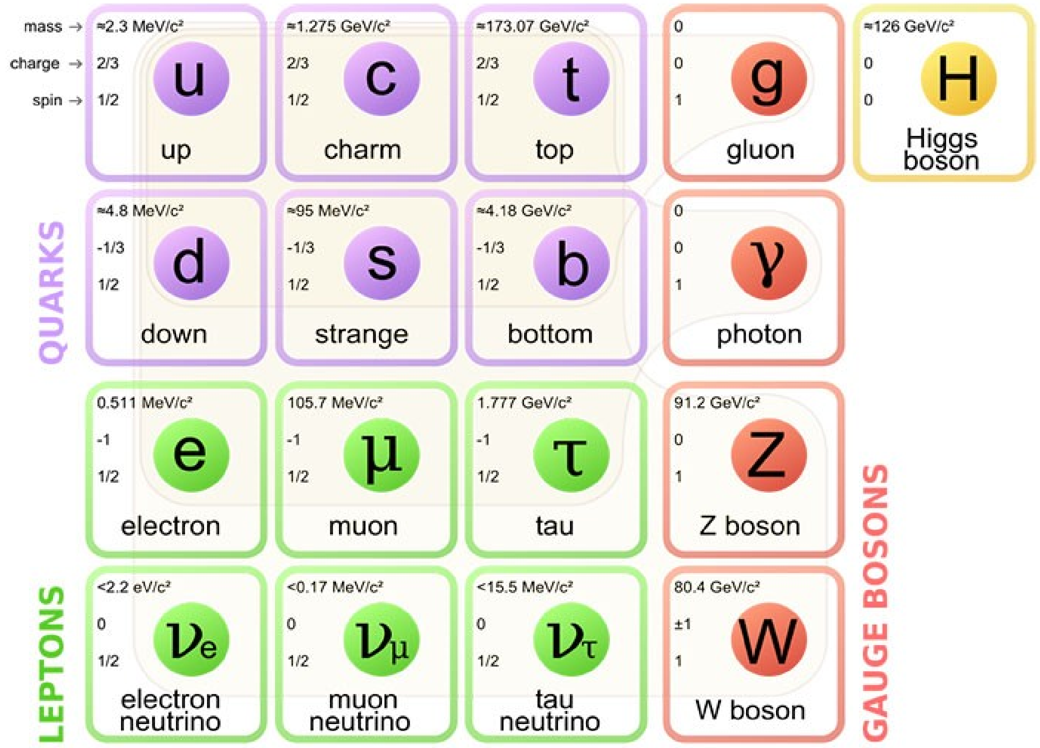
Standard Model of Particle Physics

Describes
fundamental interactions
between
elementary particles

Gauge Theory of

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

gauge group.



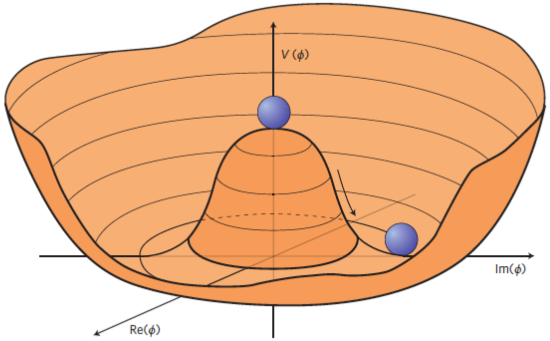
Successful Theory!

Experimentally Verified!

Standard Model of Particle Physics

The Higgs Potential acquires a vacuum

$$V(\Phi) = \mu_H^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$



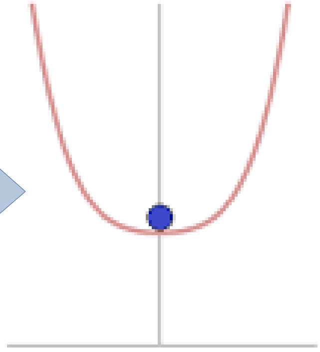
Electro-Weak
Symmetry Breaking

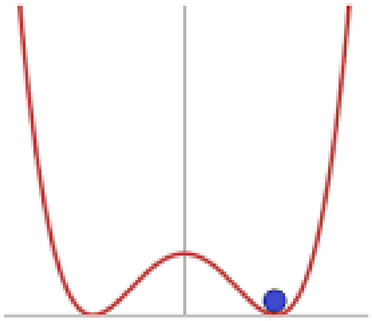
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

Higgs Field
Vaccum Expectation Value
VEV = 246 *GeV*

Fixes the EW scale

Can not break
Symmetry





Symmetry
is
broken
spontaneously

Standard Model of Particle Physics

A relevant part in the Standard Model Lagrangian is given by,

$$\mathcal{L} \supset \left|D^\mu \Phi\right|^2 - V(\Phi) + \mathcal{L}_{\text{Yukawa}}$$

where,

$$D_\mu = \partial_\mu - i g_2 \frac{\sigma^a}{2} W_\mu^a - i g_1 \frac{Y}{2} B_\mu$$

The Yukawa Lagrangian is

$$\mathcal{L}_{\text{Yukawa}} = \sum_{\text{all fermions}} \bar{\Psi}_i Y_{ij} \Psi_j \Phi$$

The Higgs Potential is

$$V(\Phi) = \mu_H^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Couplings (dimensionless)

Gauge Couplings

$$g_1, g_2, g_3,$$

Scalar quartic Coupling

$$\lambda,$$

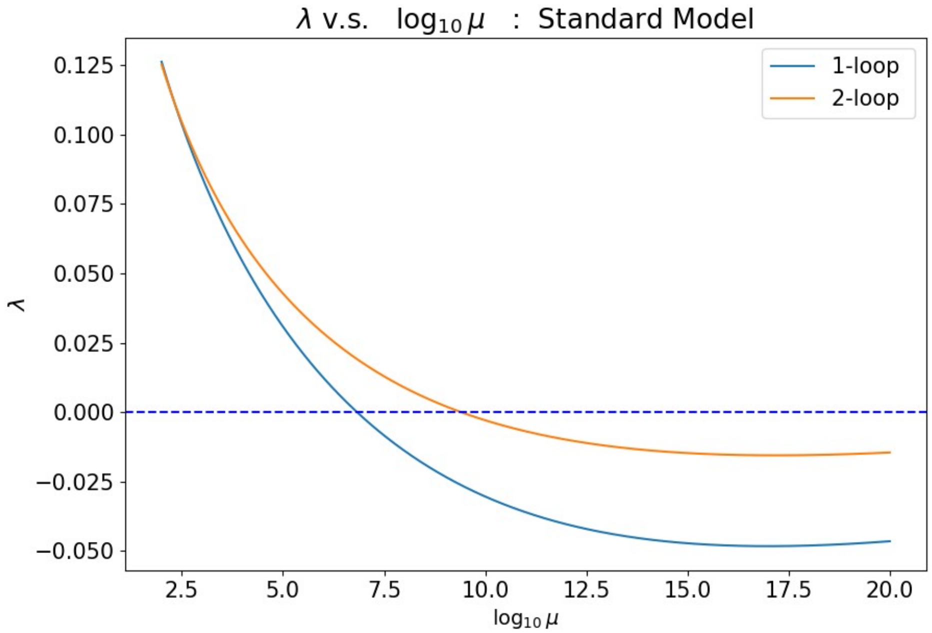
Yukawa coupling Y_t

Standard Model of Particle Physics

For SM, one-loop β-function of Higgs quartic coupling λ is

$$\beta_\lambda^{(1)} = \frac{1}{16\pi^2}\left[\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}g_1^2\lambda - 9g_2^2\lambda + 24\lambda^2 + 12\lambda Y_u^2 + 12\lambda Y_d^2 + 4\lambda Y_e^2 - 6Y_u^4 - 6Y_d^4 - 2Y_e^4\right]$$

Two-loop β-functions are even complicated.



All couplings have fixed value at electro-weak scale (reference scale).

There are no free parameters

There are no fixed points in RG evolution of any couplings.
No Landau poles either.

Scalar Extensions of Standard Model

- *Additional Quartic Couplings*
- *RG Evolution*
- *Fixed Points ?*

Motivations for Scalar Extensions

Dark Matter Models

Vacuum Stability

FOPT

Matter – Antimatter Assymetry

So on...

Inert Scalar Extensions

Inert Singlet Model (IS)

- In IS model, the SM higgs sector is extended with a complex scalar S, singlet under $SU(2)_L$ with hypercharge Y = 0. The scalar potential is,

$$V_{IS} = -\mu_h^2 \Phi^\dagger \Phi + m_S^2 S^* S + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_S (S^* S)^2 + \lambda_{HS} (\Phi^\dagger \Phi)(S^* S).$$

Inert Doublet Model (IDM)

- The Higgs sector contains two $SU(2)_L$ doublets. The scalar potential is

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

$$V_{IDM} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2$$
$$+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda_5 ((\Phi_1^\dagger \Phi_2)^2 + h.c.).$$

Inert Triplet Model (ITM)

- The SM higgs sector is extended with an $SU(2)_L$ triplet with zero hypercharge.

$$T = \frac{1}{2} \begin{pmatrix} T_0 & \sqrt{2} T^+ \\ \sqrt{2} T^- & -T^0 \end{pmatrix}$$

The scalar potential is given by

$$V_{ITM} = m_h^{2\,\dagger} \Phi + m_T^2 Tr(T^\dagger T) + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_t (Tr(T^\dagger T))^2 + \lambda_{ht} \Phi^\dagger \Phi Tr(T^\dagger T).$$

Inert Scalar Extensions

Disclaimer:

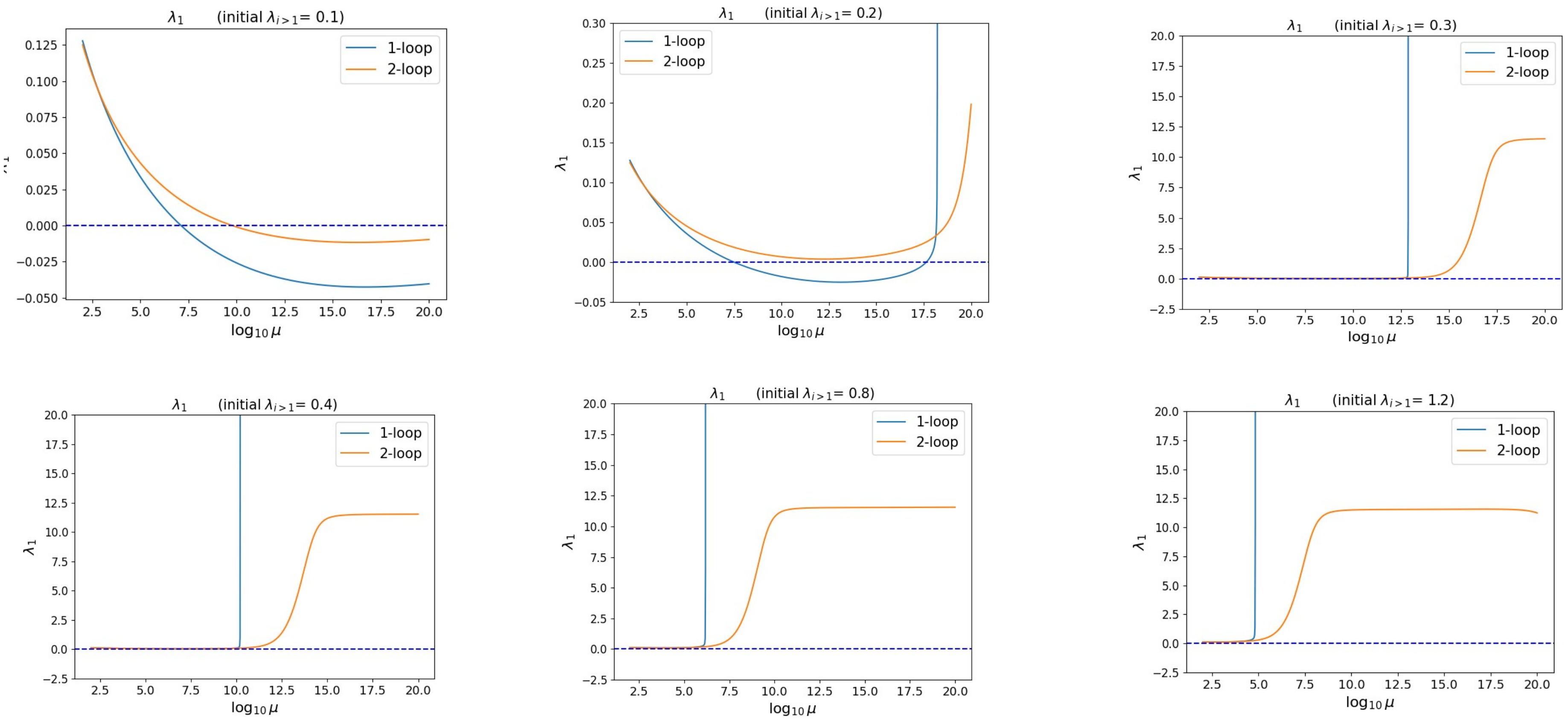
- Proper fixed points where β- functions of all couplings vanish do not appear in SM extensions.
- Instead we focus on the scalar quartic couplings.
  More specifically, the SM -like Higgs quartic coupling.
- We are interested in what sort of interactions could influenze such behaviour

Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

Inert Singlet Model (IS)

Fixed points in λ_1



Inert Scalar Extensions

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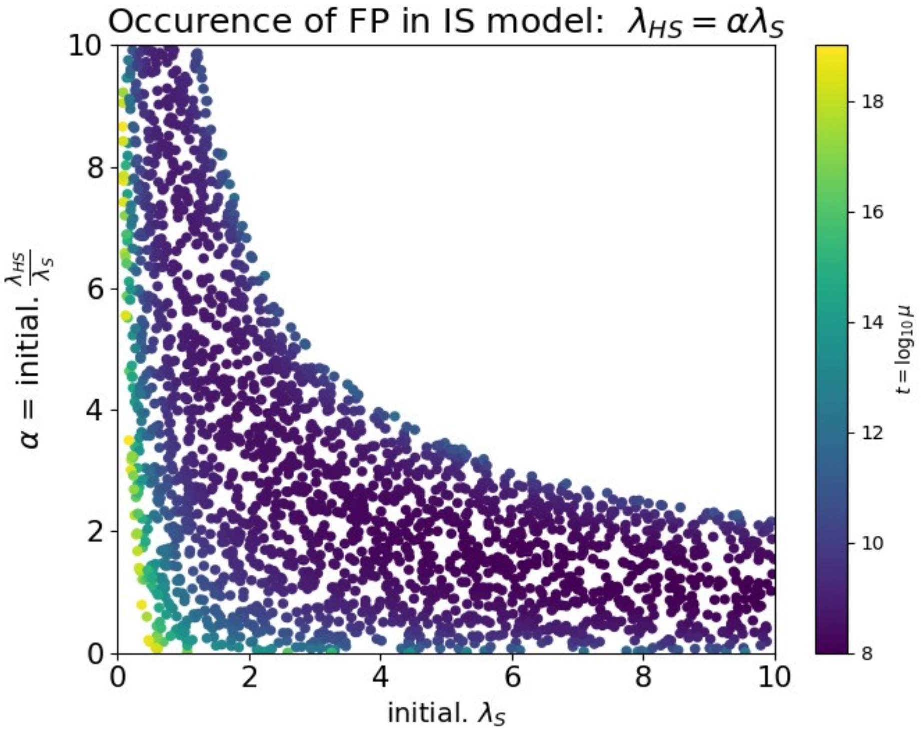
Inert Singlet Model (IS)

Fixed points in λ_1



For higher initial values,
1-loop evolution has Landau pole

$$\alpha = \frac{\text{initial } \lambda_{HS}}{\text{initial } \lambda_S}$$

For small values of λ_S, FPs occur when α is larger

For large values of λ_S, FPs get spoiled when α is larger

Could be significant in strongly coupled systems

Inert Scalar Extensions

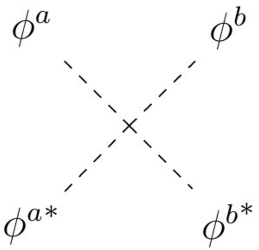
Inert Singlet Model (IS)

The λ_{HS} -term (portal term) is given by,

$$\Phi^\dagger\Phi\ S^*S = |\phi_0|^2\,|S|^2 + \left|\phi^+\right|^2|S|^2.$$

" square of absolute value "
interactions

tree-level diagrams are of the type



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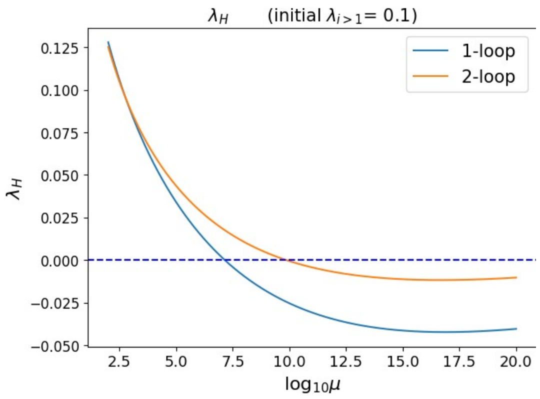
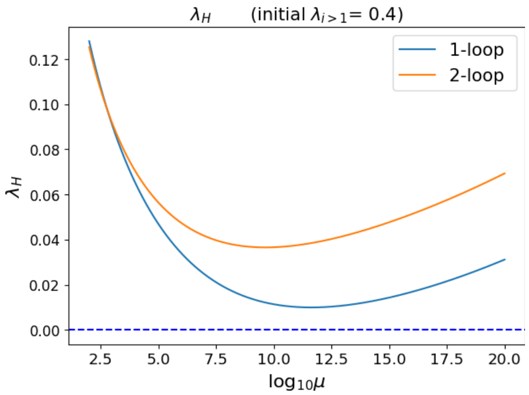
Inert Scalar Extensions

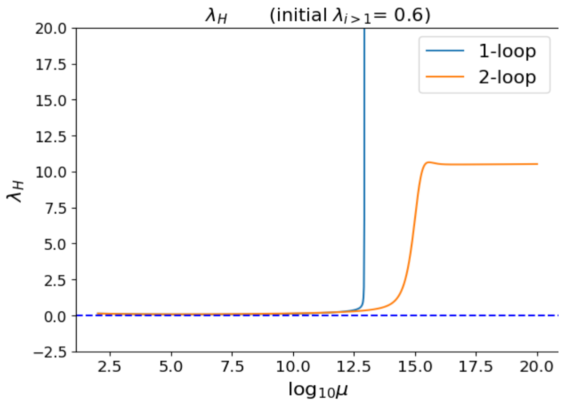
The β-functions are obtained from *SARAH 4.15.1*

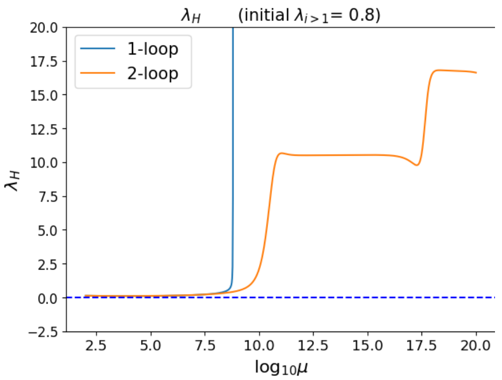
Inert Triplet Model (ITM)

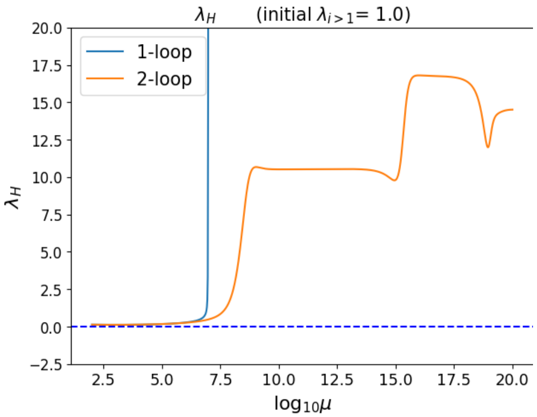
Fixed points in λ_H











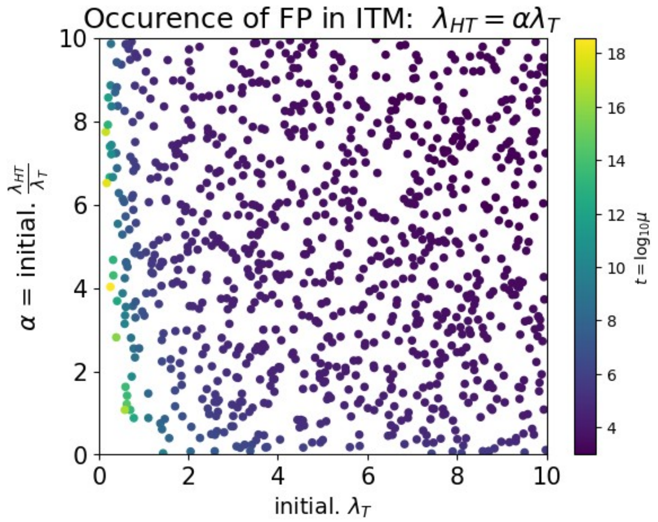
Inert Scalar Extensions

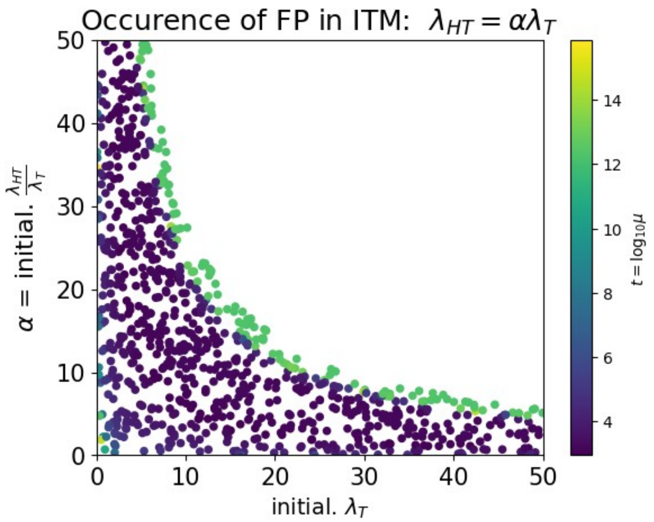
The β-functions are obtained from *SARAH 4.15.1*

Inert Triplet Model (ITM)

Fixed points in λ_H

$$\alpha = \frac{\text{initial } \lambda_{HT}}{\text{initial } \lambda_T}$$





For small values of λ_T, FPs occur when α is larger

For large values of λ_T, FPs get spoiled when α is larger

Similar to Inert Singlet !

Inert Scalar Extensions

Inert Triplet Model (ITM)

Fixed points in λ_H

$$\alpha = \frac{\text{initial } \lambda_{HT}}{\text{initial } \lambda_T}$$

The λ_{HT} -term (portal term) is given by,

$$(\Phi^\dagger\Phi)(T^\dagger T) = \frac{1}{2}\left(|T^0|^2 |\phi^0|^2 + |T^-|^2 |\phi^0|^2 + |T^+|^2 |\phi^0|^2 + |T^0|^2 |\phi^+|^2 + |T^-|^2 |\phi^+|^2 + |T^+|^2 |\phi^+|^2 \right).$$

The λ_T -term is given by,

$$((T^\dagger T))^2 = \frac{1}{4}|T^0|^4 + \frac{1}{2}|T^0|^2 |T^-|^2 + \frac{1}{4}|T^-|^4 + \frac{1}{2}|T^0|^2 |T^+|^2 + \frac{1}{2}|T^-|^2 |T^+|^2 + \frac{1}{4}|T^+|^4 .$$

" *square of absolute value* "
interactions

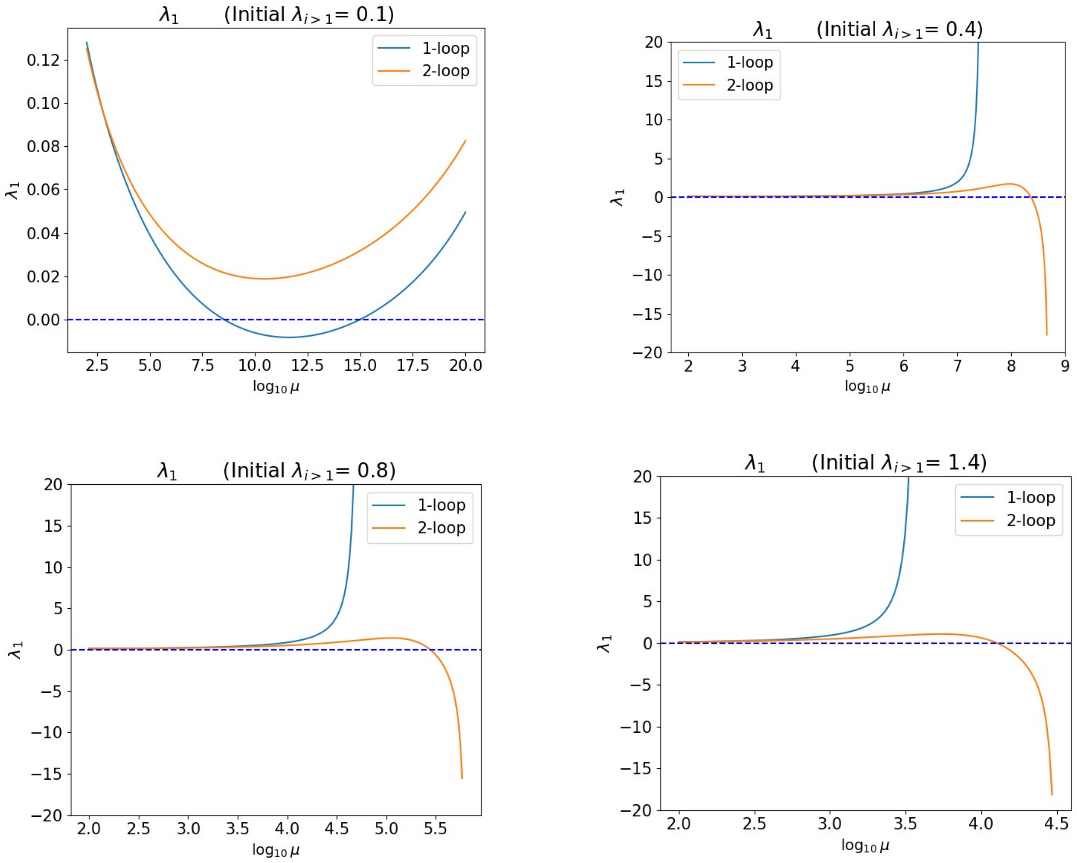
Similar to Inert Singlet !

Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

Inert Doublet Model (IDM)

Fixed points in λ_1



No Fixed Points ?

Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

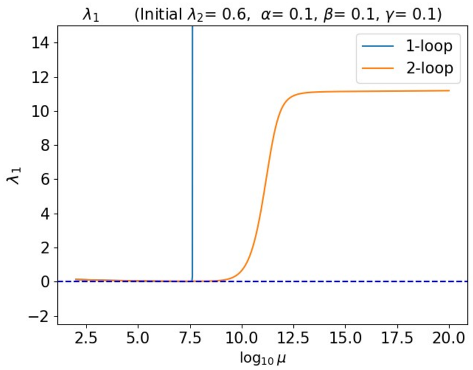
Inert Doublet Model (IDM)

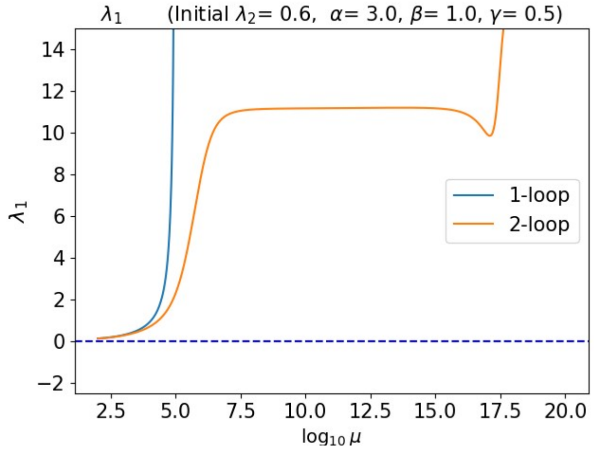
Fixed points in λ_1

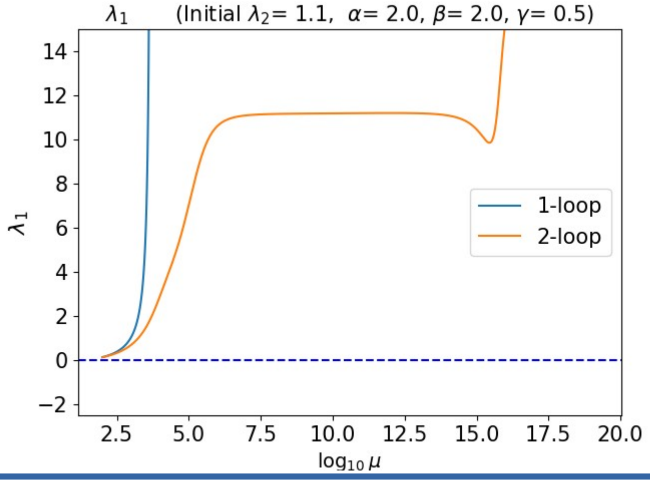
$$\alpha = \frac{\text{initial } \lambda_3}{\text{initial } \lambda_2}$$

$$\beta = \frac{\text{initial } \lambda_4}{\text{initial } \lambda_2}$$

$$\gamma = \frac{\text{initial } \lambda_5}{\text{initial } \lambda_2}$$







Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

Inert Doublet Model (IDM)

Fixed points in λ_1

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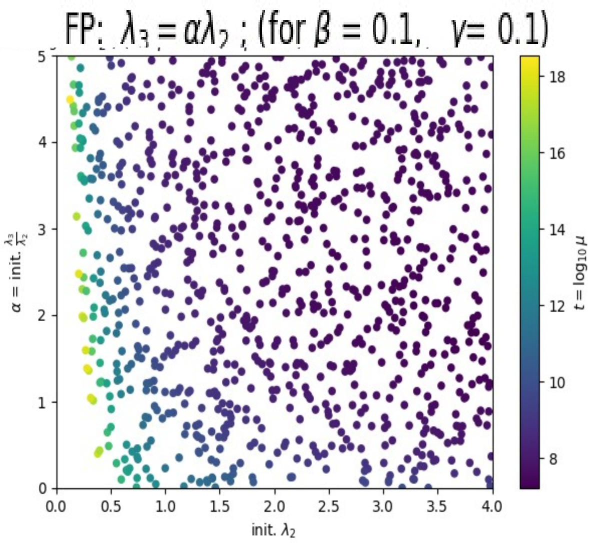
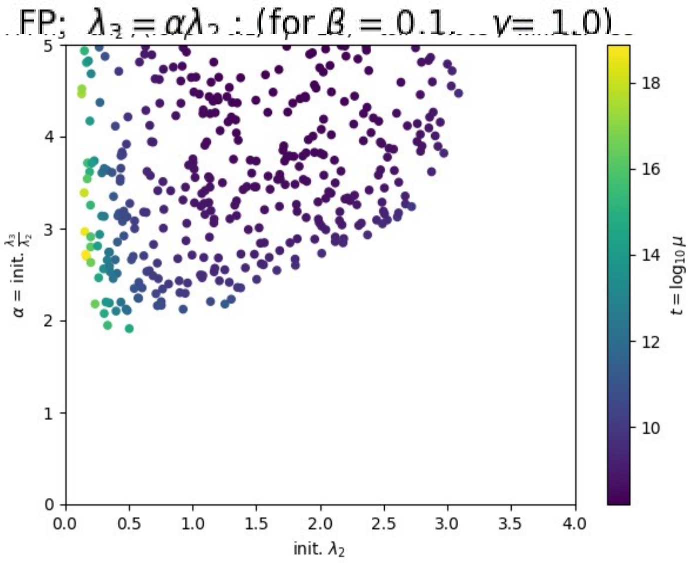




α promotes FP
γ spoils FP

Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

Inert Doublet Model (IDM)

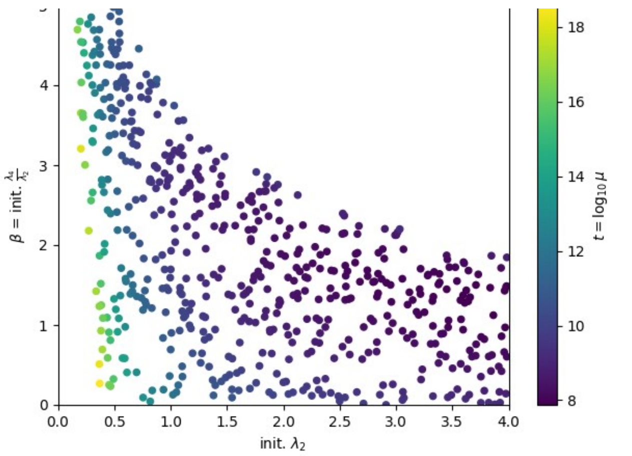
Fixed points in λ_1

$$\alpha = \frac{\text{initial } \lambda_3}{\text{initial } \lambda_2}$$

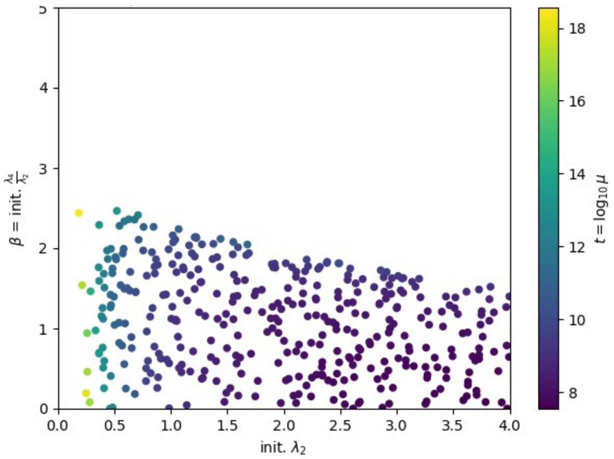
$$\beta = \frac{\text{initial } \lambda_4}{\text{initial } \lambda_2}$$

$$\gamma = \frac{\text{initial } \lambda_5}{\text{initial } \lambda_2}$$

FP: $\lambda_4 = \beta\lambda_2$; (for $\gamma = 0.1$, $\alpha = 0.1$)



FP: $\lambda_4 = \beta\lambda_2$; (for $\gamma = 0.5$, $\alpha = 1.5$)



β promotes FP for
small γ, large α

Inert Scalar Extensions

The β-functions are obtained from *SARAH 4.15.1*

Inert Doublet Model (IDM)

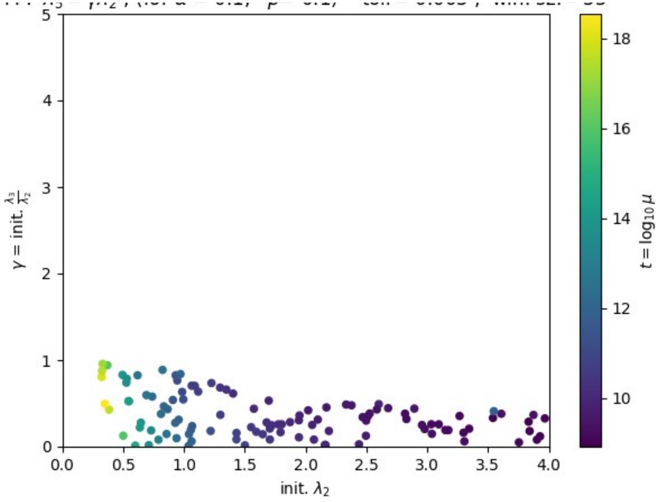
Fixed points in λ_1

$$\alpha = \frac{\text{initial } \lambda_3}{\text{initial } \lambda_2}$$

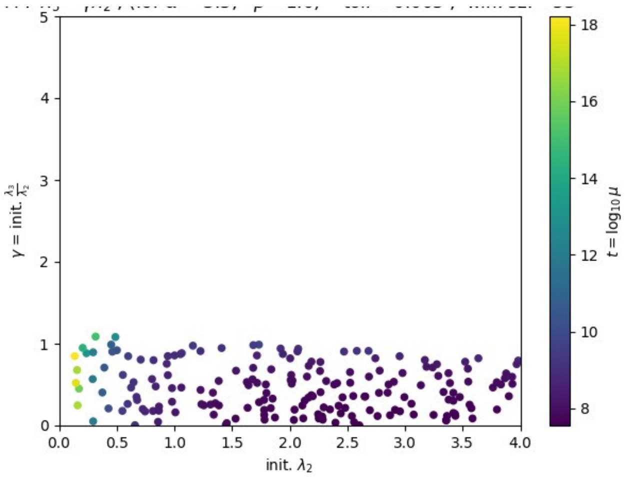
$$\beta = \frac{\text{initial } \lambda_4}{\text{initial } \lambda_2}$$

$$\gamma = \frac{\text{initial } \lambda_5}{\text{initial } \lambda_2}$$

FP: $\lambda_5 = \gamma\lambda_2$; (for α = 0.1, β= 0.1)



FP: $\lambda_5 = \gamma\lambda_2$; (for α = 3.5, β= 1.0)



γ spoils FP heavily

Inert Scalar Extensions

Inert Doublet Model (IDM)

Fixed points in λ_1

$$\phi_1^+ = |\phi_1^+| e^{i\theta_1^+} \quad \phi_1^0 = |\phi_1^0| e^{i\theta_1^0} \quad \phi_2^+ = |\phi_2^+| e^{i\theta_2^+} \quad \phi_2^0 = |\phi_2^0| e^{i\theta_2^0},$$

The λ_3 -term (α term) is given by,

$$(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) = |\phi_1^0|^2 |\phi_2^0|^2 + |\phi_1^+|^2 |\phi_2^0|^2 + |\phi_1^0|^2 |\phi_2^+|^2 + |\phi_1^+|^2 |\phi_2^+|^2$$

The λ_4 -term (β term) is given by,

$$(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) = |\phi_1^0 \phi_2^0|^2 + |\phi_1^+ \phi_2^+|^2 + 2|\phi_1^0 \phi_1^+ \phi_2^0 \phi_2^+| \cos(\theta_1^0 - \theta_1^+ - \theta_2^0 + \theta_2^+)$$

“ square of absolute value ” *“ complex phase dependent ”*

The λ_5 -term (γ term) is given by,

$$((\Phi_1^\dagger \Phi_2)^2 + \text{ h.c.}) = 2|\phi_1^0|^2 |\phi_2^0|^2 \cos(2\theta_1^0 - 2\theta_2^0) + 2|\phi_1^+|^2 |\phi_2^+|^2 \cos(2\theta_1^+ - 2\theta_2^+)$$
$$+ 4|\phi_1^0 \phi_1^+ \phi_2^0 \phi_2^+| \cos(\theta_1^0 + \theta_1^+ - \theta_2^0 - \theta_2^+).$$

“ complex phase dependent ”

Inert Scalar Extensions

If the portal coupling interactions depends only on square of absolute values of fields, then FP is promoted

If the portal coupling interactions depends on comple phases of fields, then FP is discouraged

Further observations (not explained in talk):

Increase in number of fields could help achieve the FP at lower energy scales

Some fixed points occur outside the perturbative bounds for couplings.

Evolution of Yukawa also has FP for certain parameter values.

Conclusions and Notes

- SM and its scalar extensions are **not** scale invariant theories.
- However, scalar extensions can have some parameter region where a subset of couplings have fixed point behaviour.
- The portal interactions play a role in the occurance of fixed point.

FUTURE GOALS:

- One can try more analytical (if possible non-perturbative) methods to see if fixed points occur in the evolution of couplings w.r.t. energy scale.
- Study the connections to higher symmetries like conformal symmetries.
- What sort of particle interactions cause the absence of Conformal field theories in nature.

THANK YOU

FOR YOUR

ATTENTION

Back Up

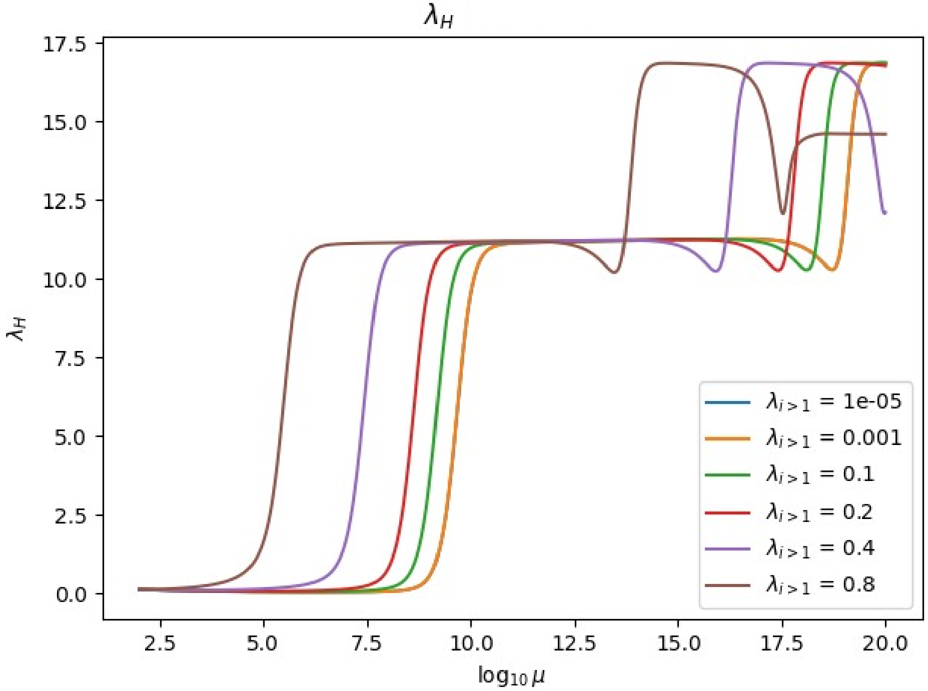
Higher Scalar Multiplets

The β-functions are obtained from *SARAH 4.15.1*

Inert Quintuplet Model (5-plet)

Fixed points in λ_1

$$V = m_H^2 \Phi^\dagger \Phi + m_\eta^2 \eta^\dagger \eta + \lambda_h (\Phi^\dagger \Phi)^2 + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_{H\eta} \Phi^\dagger \Phi \eta^\dagger \eta.$$



$$\eta = \begin{pmatrix} \eta^{+++} & \eta^{++} & \eta^{+} & \eta^{0} & \eta^{-} \end{pmatrix}^T$$

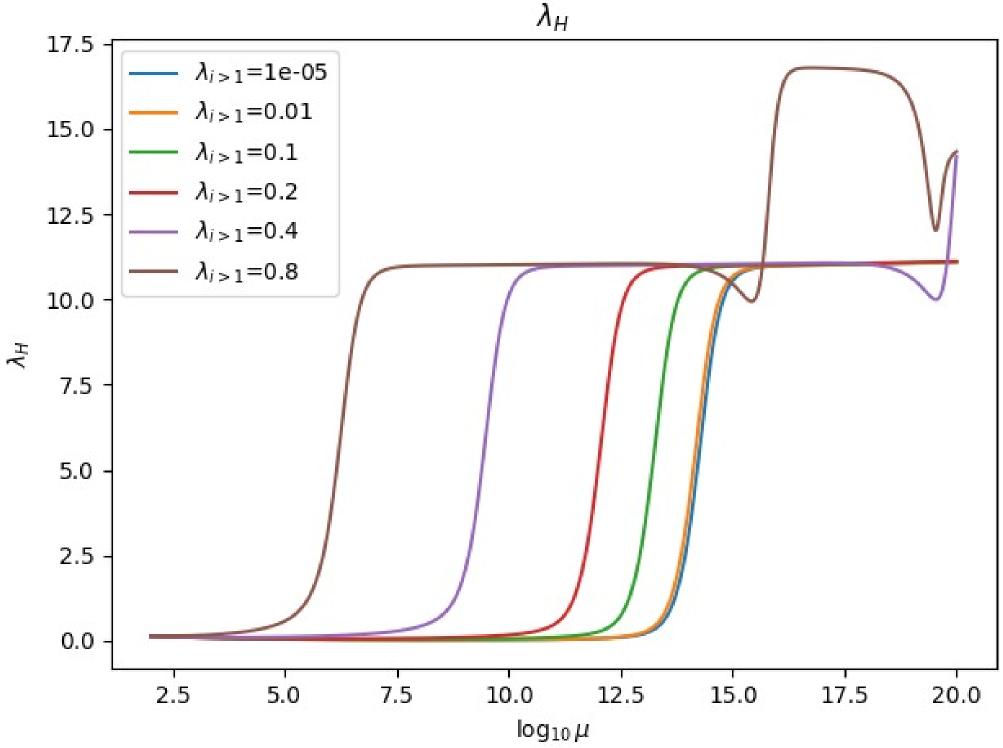
Higher Scalar Multiplets

The β-functions are obtained from *SARAH 4.15.1*

Inert Quadruplet Model (4-plet)

Fixed points in λ_1

$$V = m_H^2 \Phi^\dagger \Phi + m_\chi^2 \chi^\dagger \chi + \lambda_h (\Phi^\dagger \Phi)^2 + \lambda_\chi (\chi^\dagger \chi)^2 + \lambda_{H\chi} \Phi^\dagger \Phi \chi^\dagger \chi.$$



$$\chi = \left(\chi^{+++} \quad \chi^{++} \quad \chi^{+} \quad \chi^{0}\right)^T$$

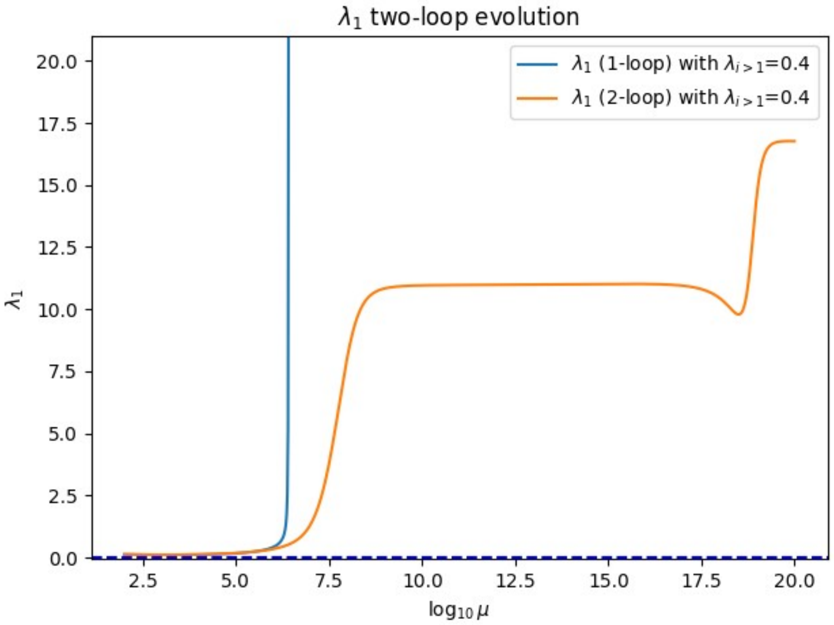
Non-Inert Models

The β-functions are obtained from *SARAH 4.15.1*

Type – II SEESAW

Fixed points in λ_1

$$V = -m_H^2 \Phi^\dagger \Phi + \lambda_1 (\Phi^\dagger \Phi)^2 + m_T^2 \ \mathrm{Tr}[T^\dagger T] + \lambda_2 (\Phi^\dagger \Phi) \mathrm{Tr}[T^\dagger T] + \\ + \lambda_3 \ (\mathrm{Tr}[T^\dagger T])^2 + \lambda_4 \ \mathrm{Tr}[T^\dagger T T^\dagger T] + \lambda_5 \ \Phi^\dagger T T^\dagger \Phi + \mu (\Phi^T (i\tau_2) T^\dagger \Phi + \mathrm{h.c.})$$



$$T = \begin{pmatrix} \frac{t^+}{\sqrt{2}} & t^{++} \\ t^0 & -\frac{t^+}{\sqrt{2}} \end{pmatrix} .$$

Perturbativity bounds on Parameters

Any perturbative expansion of amplitudes or cross-sections is valid only if the expansion parameter is less than unity.

This Perturbative Unitarity puts restrictions on the values on the parameters as follows

$$|\lambda_i| \leq 4\pi, \quad |g_i| \leq 4\pi, \quad |Y_i| \leq \sqrt{4\pi}$$

N. Haba, H. Ishida, N. Okada, Y. Yamaguchi

Eur.Phys.J.C 76 (2016) 6, 333

Perturbativity bounds: Conclusions

The First Order Phase Transitions prefers λ_i greater than unity.

But **Perturbativity could restrict the parameter space to a very small region**.

More constraints on parameter range can be obtained by studying vacuum stability.

This may rule out the model for FOPT or could put very strict constraint on the parameter range allowed.

Future Studies may include understanding the stability of vacuum in presence of finite temperature, the phase transitions in early universe etc.

Coleman-Weinberg Effective Potential

PHYSICAL REVIEW D VOLUME 7, NUMBER 6 15 MARCH 1973

Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

Sidney Coleman
and
Erick Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 8 November 1972)

We investigate the possibility that radiative corrections may produce spontaneous symmetry breakdown in theories for which the semiclassical (tree) approximation does not indicate such breakdown. The simplest model in which this phenomenon occurs is the electrodynamics of massless scalar mesons. We find (for small coupling constants) that this theory more closely resembles the theory with an imaginary mass (the Abelian Higgs model) than one with

singularity in the coupling constant. However, as we shall show immediately, this singularity is illusory; it is eaten by the renormalization counterterms.]

Of course, the integral in Eq. (3.3) is still ultra-

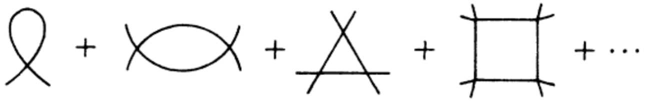
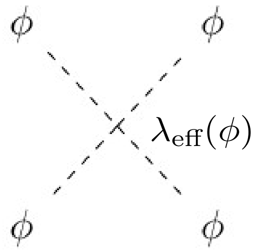


FIG. 2. The one-loop approximation for the effective potential.

Instead of perturbative series, series in loops are considered

$$V = \frac{\lambda}{4!}\phi^4$$

$$\begin{aligned} V &= \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2}\left(\ln\frac{\phi}{M^2} - \frac{25}{6}\right) \\ &= \left[\frac{\lambda}{4!} + \frac{\lambda^2}{256\pi^2}\left(\ln\frac{\phi}{M^2} - \frac{25}{6}\right)\right]\phi^4 \\ &\simeq \lambda_{\rm eff}(\phi)\phi^4 \end{aligned}$$



Coleman-Weinberg Effective Potential

For Standard Model, we have the RG improved Coleman-Weinberg effective potential as,

$$V_{\text{eff}}(h,\mu) \simeq \lambda_{\text{eff}}(h,\mu)\frac{h^4}{4} \quad \text{for} \quad h \gg v$$

where

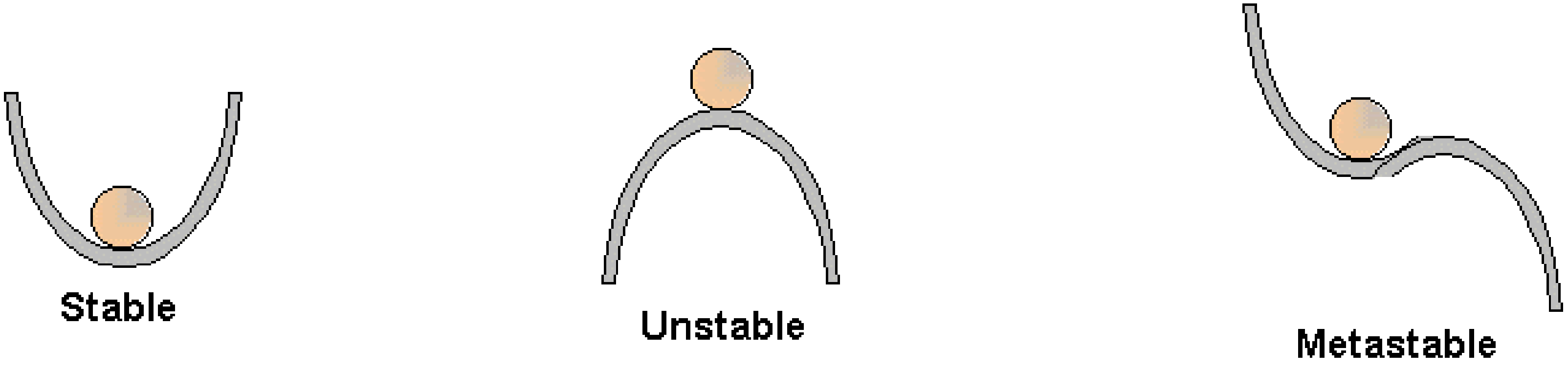
$$\lambda_{\text{eff}}(h,\mu) \simeq \lambda_h(\mu) + \frac{1}{16\pi^2} \sum_{\substack{i=W^\pm,Z,t,\\ h,G^\pm,G^0}} n_i \kappa_i^2 \left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]$$

We need additional scalars for improving stability at Planck scale

One such model is the Type-II Seesaw model

Stability of Vacuum

Stability, Instability, Metastability

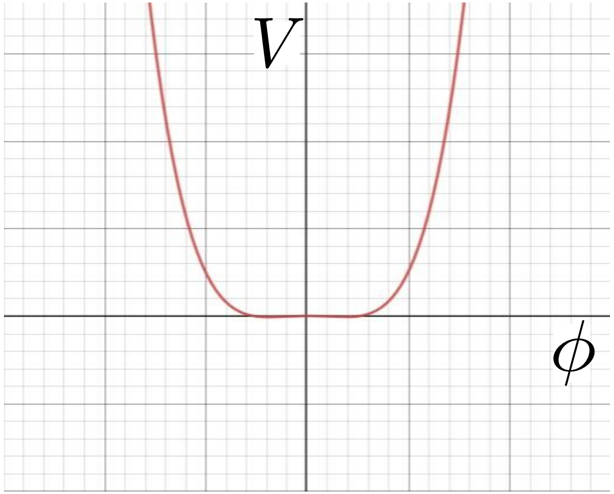


<u>Stability</u>

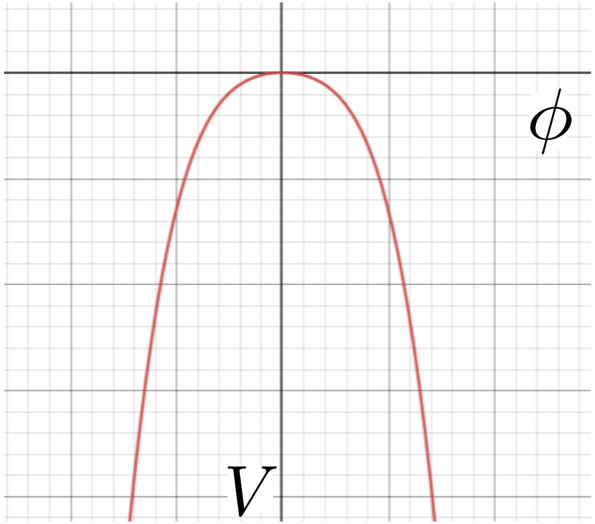
(naively) “the ability of system to come back to its original state”

Stability of the Potential

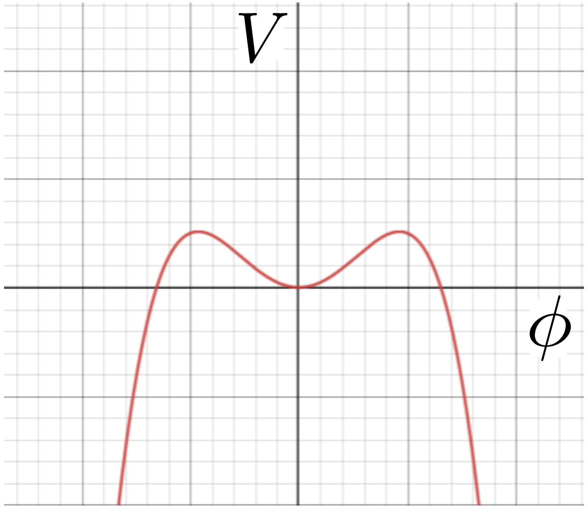
$$V(\Phi) = \mu_H^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$



$\mu_H^2 > 0, \quad \lambda > 0$



$\mu_H^2 < 0, \quad \lambda < 0$



$\mu_H^2 > 0, \quad \lambda < 0$

Stability of the Potential

$$\mu \frac{d}{d\mu}\lambda(\mu) = \beta(\lambda)$$

The fermion couplings to Higgs give negative contributions.

$$\mathcal{L} \sim Y_t \bar{Q}\Phi t_R \quad \Longrightarrow \quad \mu\frac{d\lambda}{d\mu} \approx -\frac{3}{8\pi^2}Y_t^4$$

On solving, we get

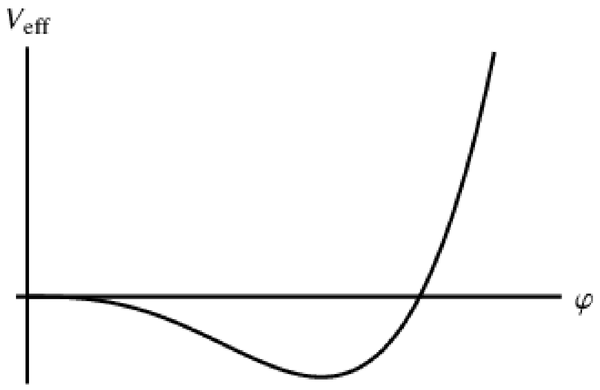
$$\lambda(\mu) = \lambda - \frac{3}{8\pi^2}Y_t^4 \ln(\frac{\mu}{v})$$

For $m_h^2 > \frac{3m_t^2}{\pi^2 v^2}\ln(\frac{\Lambda}{v})$ we get $\lambda(\mu) < 0$

Stability is in question at higher energy scales !

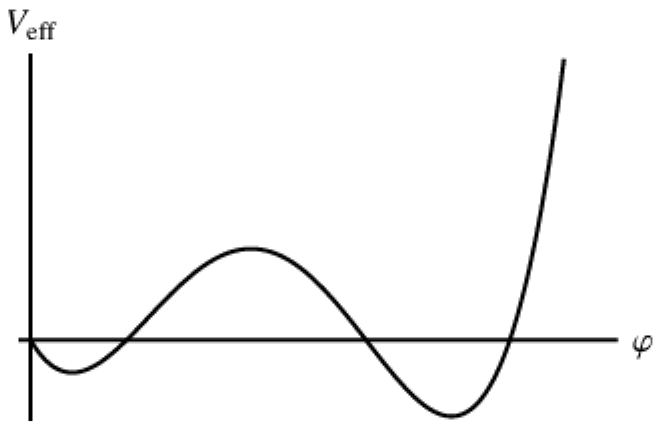
Stability of the Potential

Stable



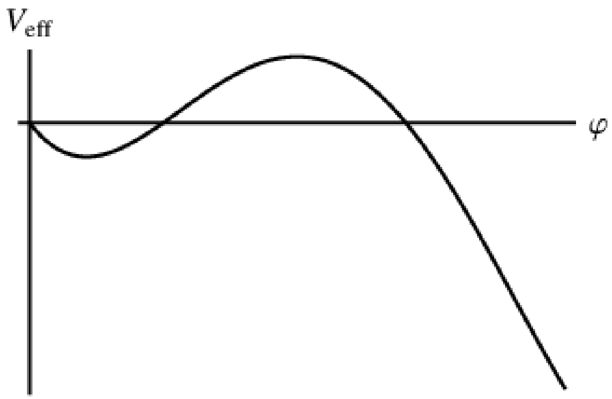
$$\lambda_{\text{eff}} > 0$$

Metastable



$$0 > \lambda_{\text{eff}}(\mu) \gtrsim \frac{-0.065}{1 - 0.01\log(\frac{v}{\mu})}$$

Unstable



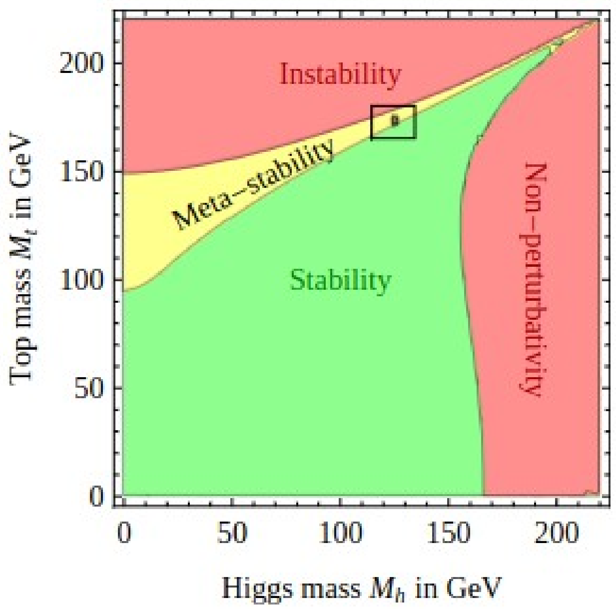
$$\lambda_{\text{eff}}(\mu) \lesssim \frac{-0.065}{1 - 0.01\log(\frac{v}{\mu})}$$

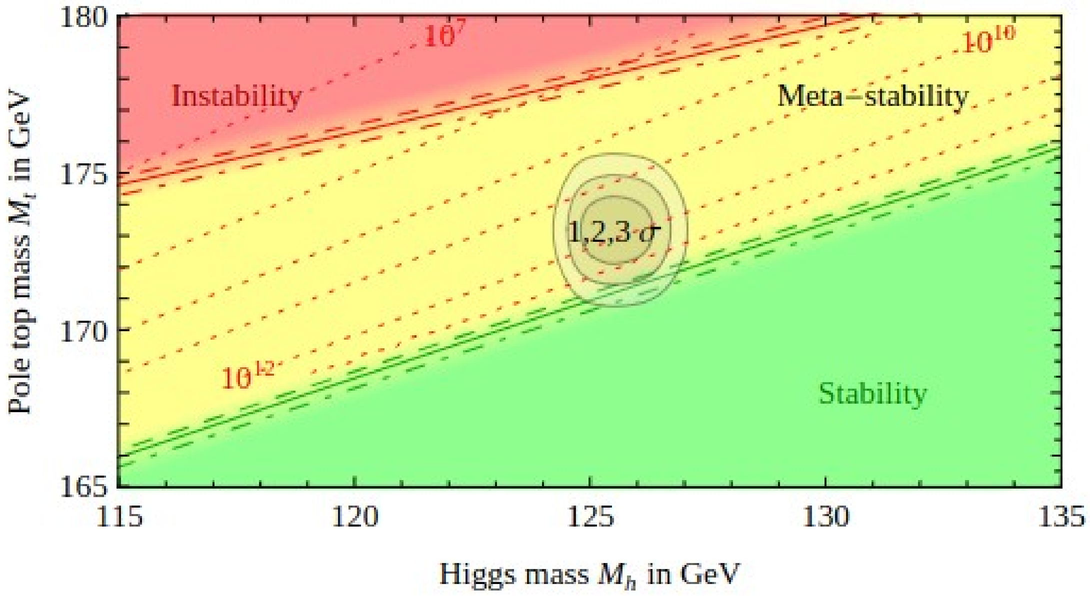
G. Isidori et. al.: NPB 609 (2001) 387

Stability of the Potential

“If the SM is extrapolated with its measured couplings up to the Planck scale, then the Higgs vacuum sits very close to the border between stable and metastable within 1.3 standard deviations of being stable”

Bass, S.D., De Roeck, A. & Kado, M
Nat Rev Phys 3, 608–624 (2021)





Degrassi et. al. :JHEP 1208, 098 (2012)

Standard Model (Hypothetical Scenario)

Lets say, $\lambda = 0.3$