Primordial Black Holes from Supercooled Phase Transitions with Radiative Symmetry Breaking

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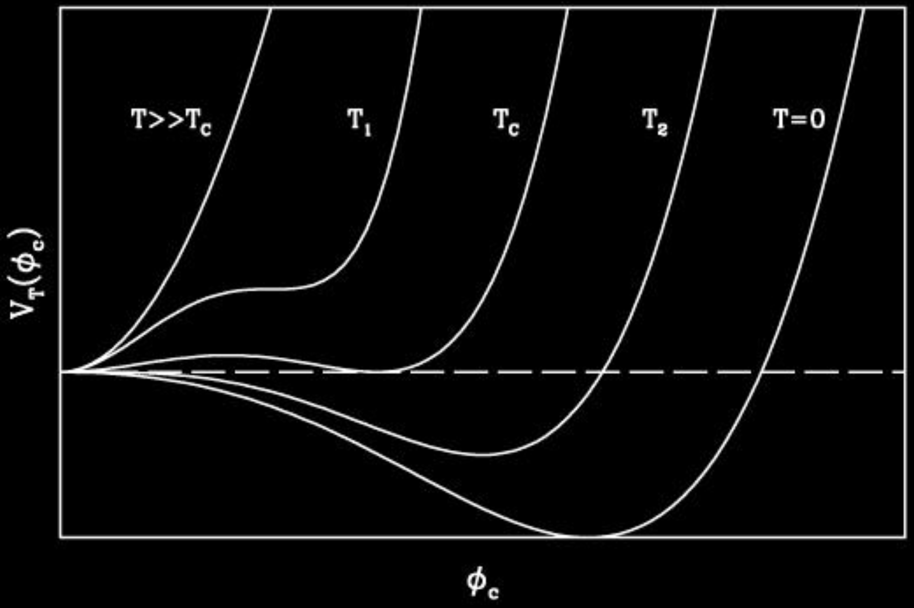
From Big Bang to Now: a Theory-Experiment Dialogue
SRM University- Andhra Pradesh

First Order Phase Transitions



$$V_T(\phi_c) = V_{\text{tree}}(\phi_c) + V_{1-\text{loop}}(\phi_c) + V_{\text{thermal}}(\phi_c, T)$$

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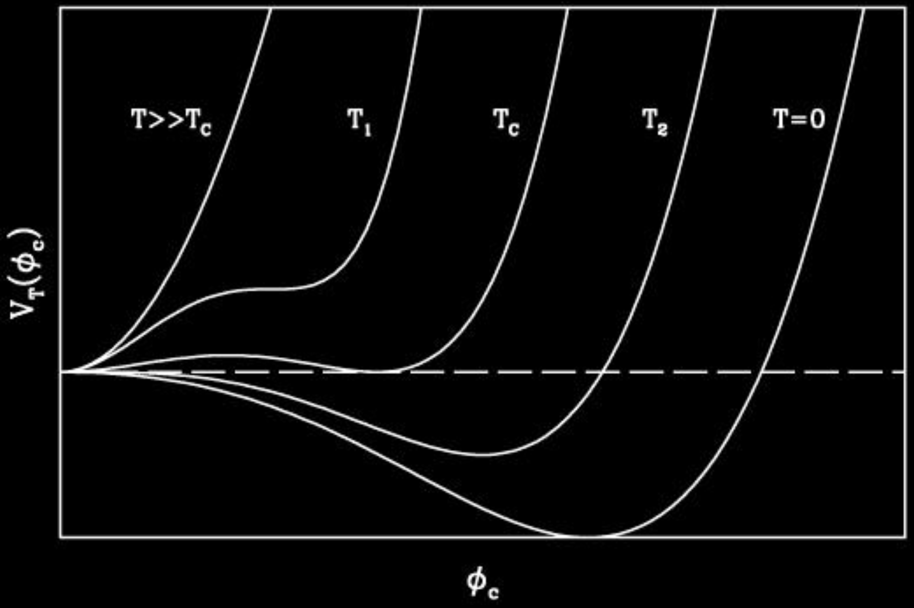

Tree level potential

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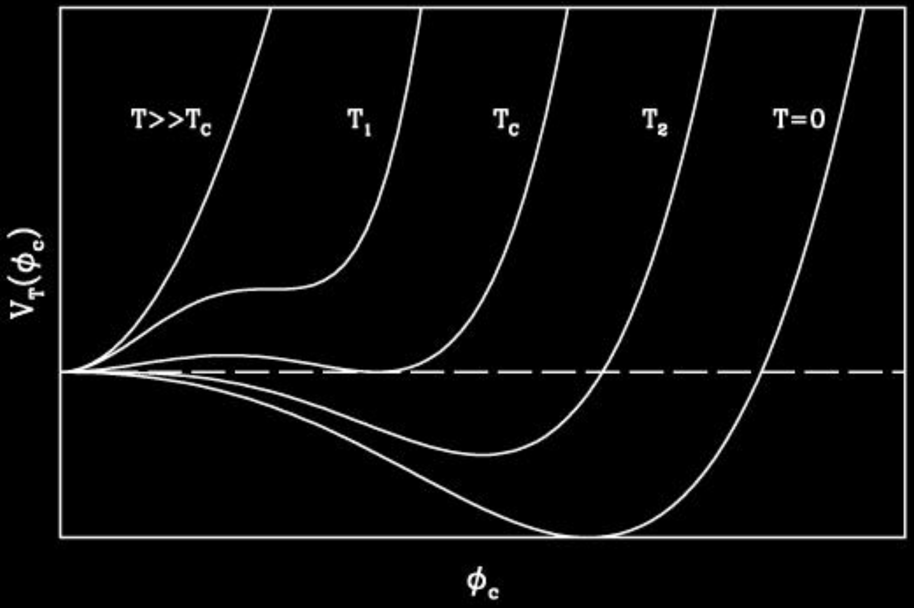

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1-loop corrections (radiative)

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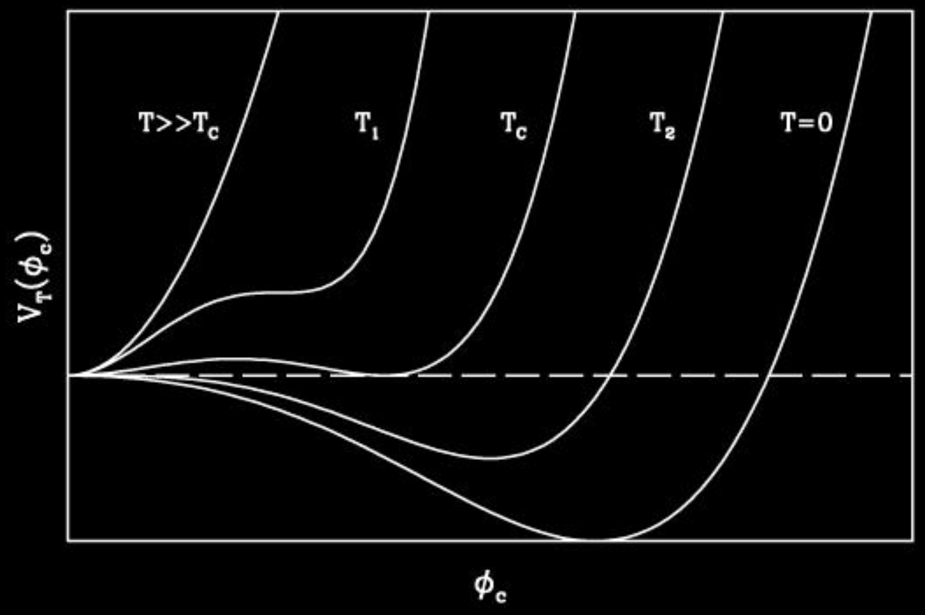

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Finite Temperature Corrections

$$V_T(\phi_c) = V_{\mathrm{tree}}(\phi_c) + V_{1-\mathrm{loop}}(\phi_c) + V_{\mathrm{thermal}}(\phi_c, T)$$

FOPT from Radiative Symmetry Breaking

$$\frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d \longrightarrow \text{Tree level potential}$$

2307.04694

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Flat direction

$$\phi_a = \nu_a\chi, \;\; \nu_a\nu_a = 1$$

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$$V(\chi) = \frac{\lambda_\chi(\mu)}{4}\chi^4, \qquad \left(\lambda_\chi(\mu) \equiv \frac{1}{3!}\lambda_{abcd}(\mu)\nu_a\nu_b\nu_c\nu_d\right)$$

Renormalization Scale

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$$V_q(\chi) = \frac{\bar\beta}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4 \quad \text{with} \quad \bar\beta \equiv \left[\mu\frac{d\lambda_\chi}{d\mu}\right]_{\mu=\tilde\mu} \quad \text{and} \quad \lambda_\chi(\tilde\mu) = 0$$

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FOPT from Radiative Symmetry Breaking

Including finite temperature corrections, under supercooled expansion

$$\bar{V}_{\rm eff}(\chi, T) = \frac{m^2(T)}{2}\chi^2 - \frac{k(T)}{3}\chi^3 - \frac{\lambda(T)}{4}\chi^4 + \ldots$$

where

$$m^2(T) \equiv \frac{g^2 T^2}{12}, \qquad k(T) \equiv \frac{\tilde{g}^3 T}{4\pi}, \qquad \lambda(T) \equiv \bar{\beta} \log \frac{\chi_0}{T},$$

$$g^2 \equiv \sum_b n_b m_b^2(\chi)/\chi^2 + \sum_f m_f^2(\chi)/\chi^2, \qquad \tilde{g}^3 \equiv \sum_b n_b m_b^3(\chi)/\chi^3$$

also $\epsilon \equiv \dfrac{g^4}{6\bar{\beta}\log\frac{\chi_0}{T}}$ $\longrightarrow$ measure of the supercooled expansion

2307.04694

FOPT : Important Parameters

- Nucleation Temperature:

$$N(T_n) = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1 \text{ and } \quad \Gamma(T) = \left(\frac{S_3}{2\pi T}\right)^{3/2} T^4 e^{-S_3/T}$$

- Strength of the FOPT:

- Inverse duration:

- Reheating Temperature:

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$$T_{\text{eq}} = T_n \alpha^{1/4}$$

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- Nucleation Temperature:

$$T_n \approx \chi_0 \exp\left(\frac{\sqrt{c'^2 - 16a} - c'}{8}\right) \quad a \equiv \frac{c_3 g}{\sqrt{12\bar{\beta}}}, \quad c' \equiv 4\log\frac{4\sqrt{3}\bar{M}_P}{\sqrt{\bar{\beta}}\,\chi_0}$$

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$$S_3 \approx 27\pi m^3 \frac{1+\exp(-k/(m\sqrt{\lambda}))}{2k^2 + 9\lambda m^2}$$

- Strength of the FOPT:

$$\alpha = \left(\frac{\Delta V}{\rho_r}\right)_{T=T_n} \longrightarrow \alpha \gg 1$$

Falls out of calculation

- Inverse duration:

$$\beta/H = T_n \frac{d}{dT}(S_3/T)|_{T=T_n} \longrightarrow \frac{\beta}{H_n} \approx \frac{a}{\log^2(\chi_0/T_n)} - 4$$

- Reheating Temperature:

$$T_{\text{eq}} = T_n \alpha^{1/4}$$

$$T_{\text{eq}}^4 = \frac{15\bar{\beta}\chi_0^4}{8\pi^2 g_*(T_{\text{eq}})}$$

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  Talk by Prof. Sriramkumar
- **Primordial Black Holes :** collapse of overdense regions due to delayed vacuum decay[1], collapse of curvature perturbations created from time fluctuations of bubble nucleation[2]

[1] 2106.05637, 2212.14037, 2305.04924, 2305,04942, …
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Talk by Prof. Sriramkumar

- **Primordial Black Holes :** collapse of overdense regions due to delayed vacuum decay[1], collapse of curvature perturbations created from time fluctuations of bubble nucleation[2]

Main point of this talk!

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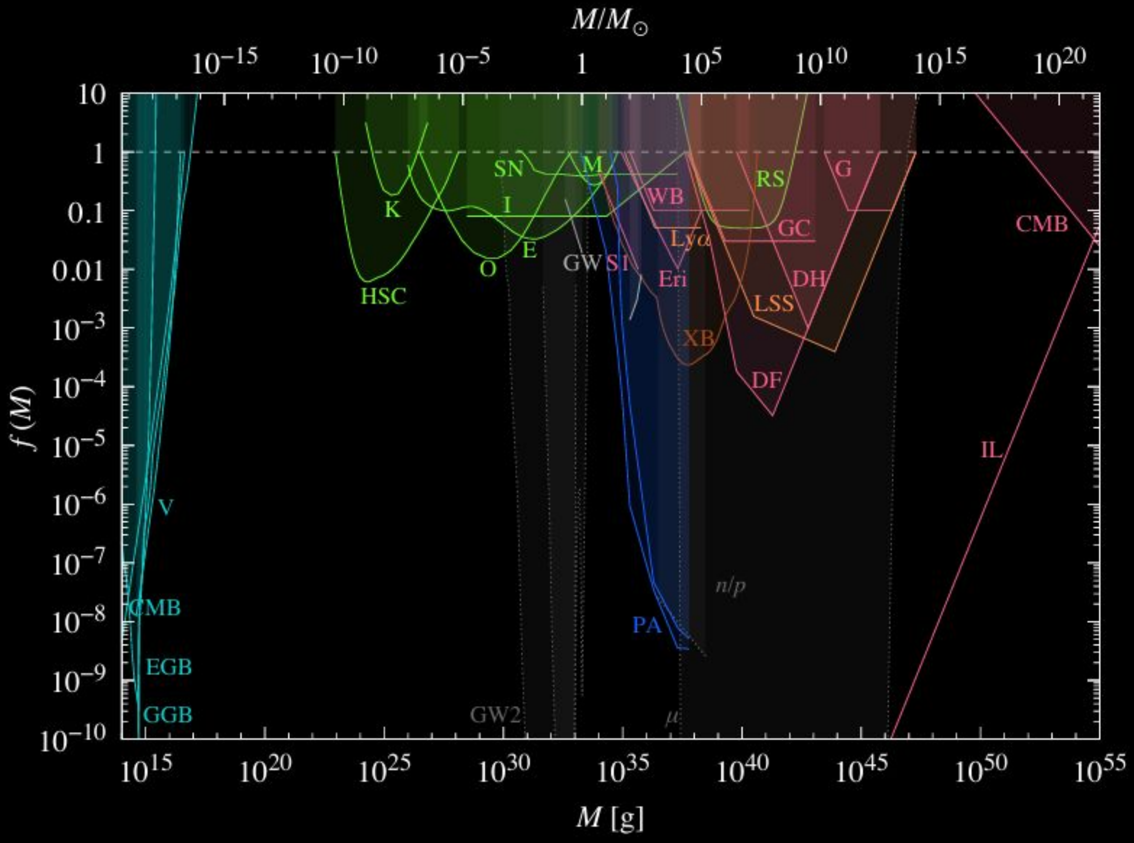
- PBH properties can help us gain insight regarding very early universe

- PBH might also be the only way for us to detect Hawking evaporation

Detour: Why PBH?



Talk by Prof. Sriramkumar

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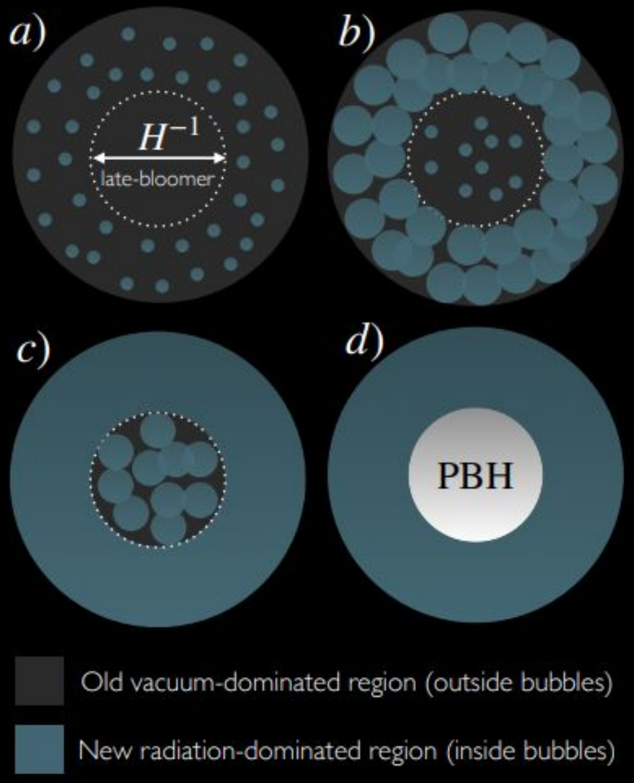
FOPT: PBH Creation



- Delayed vacuum decay in some region
- Background energy density gets diluted with the expansion of the universe
- Vacuum energy remains constant and eventually creates a overdense region
- Depending on the overdensity, it may collapse into a PBH

2305.04942

FOPT: PBH Creation

- Collapse Probability

$$\mathcal{P}_{\rm coll} \approx \exp\left[-a_{\mathcal{P}}\left(\frac{\beta}{H_n}\right)^{b_{\mathcal{P}}}(1+\delta_c)^{c_{\mathcal{P}}\frac{\beta}{H_n}}\right]$$

$$a_{\mathcal{P}} \approx 0.5646, b_{\mathcal{P}} \approx 1.266, c_{\mathcal{P}} \approx 0.6639$$

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$$f_{\mathrm{PBH}} \approx \frac{\mathcal{P}_{\mathrm{coll}}}{6.0 \times 10^{-12}} \frac{T_{\mathrm{eq}}}{500\mathrm{GeV}}$$

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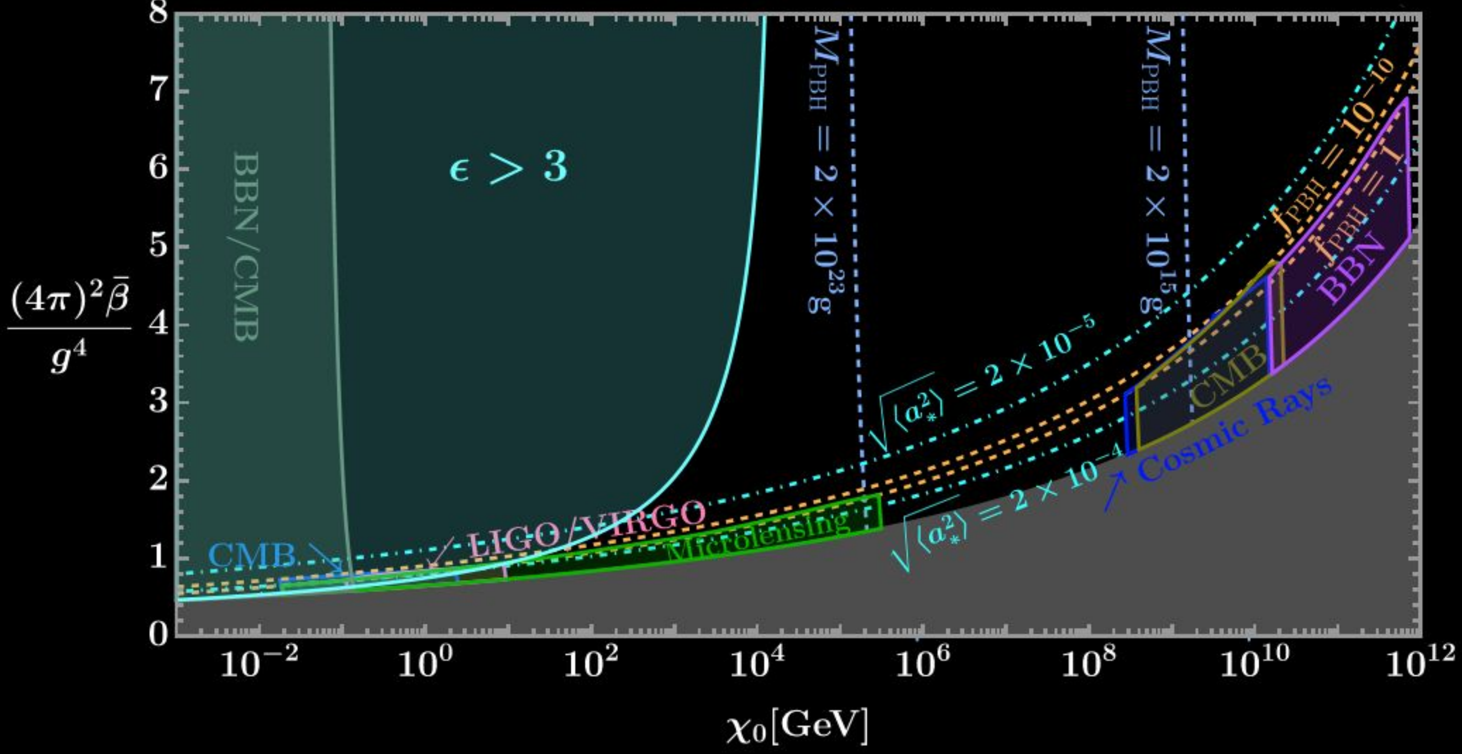
- Initial PBH spin

$$\sqrt{\langle a_*^2\rangle} \approx \frac{2.1\times 10^{-3}}{23.484 - 1.25\log_{10}(f_{\rm PBH}) - 1.25\log_{10}\left(\frac{\Omega_{\rm CDM}}{0.26}\right) - 0.625\log_{10}\left(\frac{M_{\rm PBH}}{10^{15}\ {\rm g}}\right)}$$

2409.06494, IKB, T Harada

Results

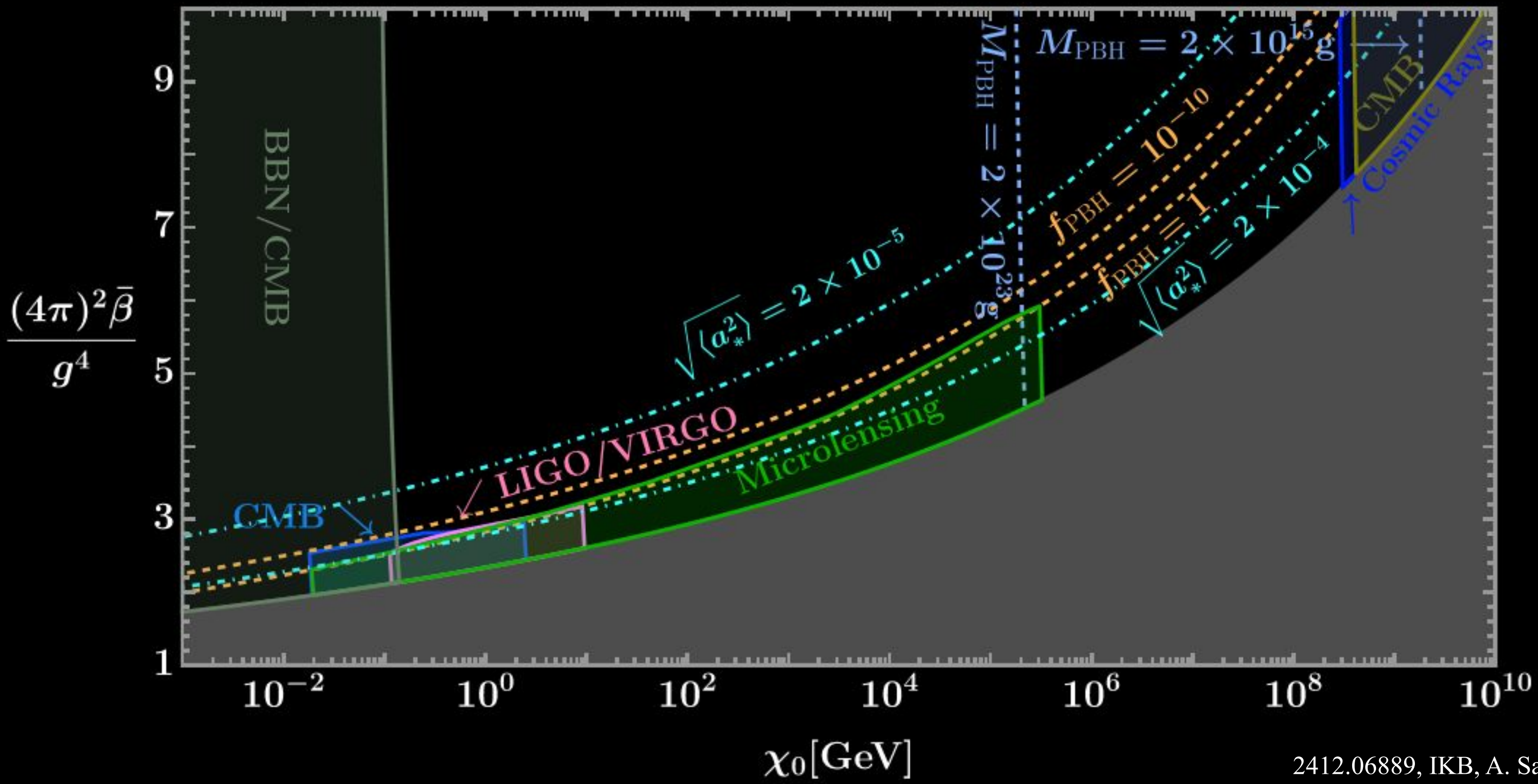
$g = \tilde{g} = 0.7$



2412.06889, IKB, A. Salvio, F. Rescigno

Results

$g = \tilde{g} = 0.5$



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Summary and Future Prospects

- RSB scenario is well motivated by various BSM extensions
- It is also the most likely scenario through which an FOPT can create PBHs
- However, this high supercooling can lead to small period of non-standard cosmology with interesting implications.
- The spin of the PBHs can also be impacted by non-standard cosmological scenario.

THANK YOU

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Renormalization Scale

One-loop correction

$$\mu\frac{dV_q}{d\mu} = 0, \qquad \text{where} \quad V_q \equiv V + V_1 + V_2 + \dots .$$

$$V_q(\chi) = \frac{\bar{\beta}}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4 \quad \text{with} \quad \bar{\beta} \equiv \left[\mu\frac{d\lambda_\chi}{d\mu}\right]_{\mu=\tilde{\mu}} \quad \text{and} \quad \lambda_\chi(\tilde{\mu}) = 0$$

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A specific example: $U(1)_{B\text{-}L}$

$$\mathscr{L}_{\rm SM}^{\rm ns} + D_\mu A^\dagger D^\mu A + \bar{N}_j i \not{D} N_j - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}$$
$$+ \left(Y_{ij} L_i \mathcal{H} N_j + \frac{1}{2} y_{ij} A N_i N_j + \text{h.c.} \right) - \lambda_a |A|^4 + \lambda_{ah} |A|^2 |\mathcal{H}|^2$$

$$D_\mu = \partial_\mu + i g_3 T^\alpha G^\alpha_\mu + i g_2 T^a W^a_\mu + i g_Y \mathcal{Y} B_\mu + i \left[g_m \mathcal{Y} + g'_1 (B - L) \right] B'_\mu$$

$$(4\pi)^2 \mu \frac{d}{d\mu} \lambda_a = 96 g'^4_1 - 48 \lambda_a g'^2_1 + 20 \lambda_a^2 + 2 \lambda_{ah}^2 + 2 \lambda_a \, \text{Tr}(y y^\dagger) - \, \text{Tr}(y y^\dagger y y^\dagger)$$

$$\bar{\beta} = \frac{96 g'^4_1}{(4\pi)^2} \qquad\qquad g = 2\sqrt{3} |g'_1|$$